

NAVAL POSTGRADUATE SCHOOL

Monterey, California



THESIS

**THE DESIGN AND OPTIMIZATION OF A POWER
SUPPLY FOR A ONE-METER ELECTROMAGNETIC
RAILGUN**

by

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December 2001

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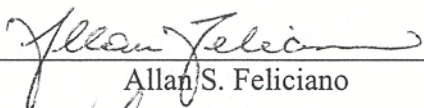
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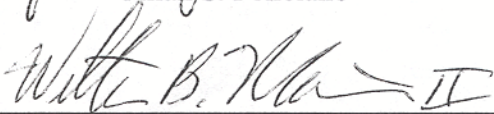
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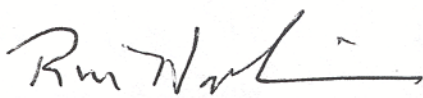
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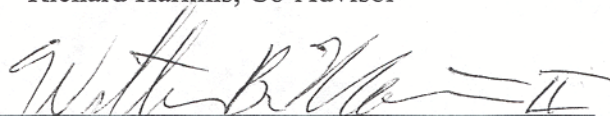
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ABSTRACT

A naval electromagnetic railgun would be a considerable asset against a littoral environment. By accelerating projectiles to 3 km/s, a naval railgun would be capable of reaching 300-400 nautical miles. Problems such as rail erosion, energy storage and fire control prevent the railgun from becoming a weapon to date. At the Naval Postgraduate School, the Physics Department continues to investigate and develop concepts to overcome these challenges. As part of the methodology, previous students built a one-meter railgun system for experimentation. The existing 1.6 mF power supply is insufficient to fire this railgun effectively. To design a sufficient power supply a MATLAB code was created to simulate a generated current pulse and to predict the subsequent railgun performance. Interrelated factors such as railgun geometry, muzzle velocity, current density and contact surface area were taken into consideration. Also, tradeoffs in capacitance, projectile mass and residual current were weighed against one another to achieve desired railgun performances. From numerous simulations, this study determined that the one-meter railgun with a 21.5 mF power supply could fire a 0.158-kg projectile at a velocity of 1 km/s, and leave a residual current of only 4% of the initial energy once the projectile exits the rails.

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I. INTRODUCTION

A. SCOPE

The scope of this thesis is to study and examine the power supply required for a pre-existing 1.2-meter railgun, while accounting for numerous factors such as muzzle velocity, railgun length, projectile mass, and current density. In addition, we intend to explore a hypothetical power supply for a 10-meter naval railgun.

B. MOTIVATION FOR A NAVAL EM RAILGUN

Today, the argument for a naval electromagnetic railgun relies upon two principles, necessity and feasibility. The former probes the question of whether or not the Navy needs to add a railgun to its current arsenal, and the latter explores the suitability of placing such a weapon onboard a naval vessel. That is to say, a naval railgun must prove to be useful in future warfare tactic and yet still fall within a platform's technological constraints, such as power supply and structural design. Therefore, although an electromagnetic railgun has the potential to revolutionize naval warfare, the practicality of such a weapon must first hold up to these issues.

1. Necessity

Although the immediate threat of the Soviet Union has diminished, the post-Cold War environment has created new challenges for the Navy. Naval operations have endeavored to maintain maritime supremacy by focusing on the littoral regions of the world (from the surf zone to the continental shelf). However, history has shown that this volatile environment has posed a formidable, and often deadly, challenge to naval operations. [1] Against a littoral backdrop, naval vessels must be ready to face numerous threats such as, surf zone mines, land-based forces, coastal defenses, anti-ship missiles, and diesel submarines. Furthermore, other factors may put ships at a disadvantage such as sensor degradation due to heavy land clutter, or restricted maneuverability due to shallow waters. Thus, in order to safely transit and operate effectively in these regions, naval forces must be able to handle the inherent difficulties of these confined and

congested waters. [2] It may be in the Navy's best interest to remain at large standoff distances away from the "constrained" waters of the littorals, and if so, the Navy should conceivably still be capable of completing its mission from a range of a hundred miles or more.

Today, the U.S. Navy employs three main types of weapons against targets ashore: manned aircraft, tactical missiles, and conventional naval guns. Although the "distancing" scenario does not prevent the Navy from using aircraft and missiles to project its power ashore from far off distances, the costs of such resources limits their usage. As a result, the Navy must also rely on its conventional guns to accomplish its mission. However, the muzzle velocity of the 5"/54 naval gun is about 0.81 km/s, which results in a range of only 12-15 nautical miles. This range falls well within the dangers of the littoral region. Therefore, in order to maintain ships at as safe distance, it may be in the Navy's interest to devise a weapon with an increased muzzle velocity and subsequent long-range capability.

A naval electromagnetic gun has the potential to fulfill the Navy's needs and interests. Electromagnetic launchers have overcome the velocity limitations of chemical propellants such as gunpowder or rocket fuel. [6] Because of friction due the atmosphere, velocities greater than 3.0 km/s may be impractical. However, within a velocity range of 2.5 - 3.0 km/s, the muzzle velocities would still be sufficient enough to carry the projectile to approximately 300-400 nautical miles, fulfilling the Navy's "long-range" requirement. [7] With a velocity of 2.5 km/s, a 60 kg projectile would then have 180 MJ kinetic energy, about 15 times the chemical energy of a high explosive 5" round. Furthermore, apart from providing a lethal round, the inert rounds could also replace the potentially hazardous and explosive rounds stored in ships' magazines. Therefore, the naval EM gun would not only appear to be a new weapon of choice due to lethality, but of ship safety as well. An electromagnetic railgun would thus enable the Navy to supplement its aircrafts and missiles by projecting significant power ashore from ships hundreds of miles offshore. [3]

2. Feasibility

With the increased attention towards long-range land attacks, the CNO's Strategic Studies Group (SSG) conducted a study on the feasibility of integrating a naval EM gun on board a naval platform. Performance parameters, such as range, weight, power, and cost were developed and taken into account. The parameters were as given in Table 1.B.1.

Range: 300 to 400 nmi
Projectile Mass: 60 to 70 kg
Barrel Length: ≤ 15 m
Muzzle Velocity: 2.5 to 3.5 km/s
Impact Velocity: 1.5 to 2.5 km/s
Impact Kinetic Energy: 67 –220 MJ
Firing Rate: 6 rounds/min
Power Usage: ~ 60 MW (at max range and rate)
Time of Flight: ~ 8 min (at max range)
Cost: $\leq \$5K$ per round

Table 1.B.1 Performance Parameters for a Hypothetical Naval EM Gun From Ref. [3]

The SSG proposed that with a little time invested in fundamental research, a naval electromagnetic railgun system could reasonably be fielded within 20 to 30 years. [3] One reason for the optimism was probably due technological advances in energy storage and materials. But in spite of other advances, the main reason for the optimistic outlook

for a railgun was probably due to the oncoming of an “all-electric” ship. There was reason to believe, that an all-electric ship could utilize an “electric gun”, in this case an EM gun. In January 2000, the Secretary of the Navy announced that the next generation Destroyer (DD21) would be designed to incorporate an electric drive and integrated power system (IPS). [9] Consequently, this “opened the door” for possibly providing the power architecture required for an EM gun. To begin with, DD21’s propulsion plant would be capable of providing at least 90 MW power. With this in mind and the IPS, DD21 could realistically tap into the power supply used for propulsion and redirect it towards other systems, such as combat systems. This concept makes the notion of a naval electromagnetic railgun appear feasible today.

In summary, a naval EM gun would be the weapon of choice for future naval platforms. It would enable naval ships to project long-range munitions ashore while maintaining safe distances well outside littoral waters. It would be an ideal weapon for the next generation of ships utilizing the all-electric integrated power system. Having enormous amounts of “available and redirected” power makes the EM gun a practical weapon. [3] In the meantime, numerous electromagnetic workshops and symposiums continue to strive towards the development of a concept demonstration or prototype. But although the railgun effort continues, advances in energy storage, material science, and solid-state devices bring the railgun one step closer to reality. Hence, although the development of an EM gun may be in its conceptual stages, the future may not be so far off from its actuality.

C. HISTORIC CHALLENGES

The notion of an electromagnetic railgun is not a new one. Early research dates back as far as 1901 when Birkeland developed the “Patent Electric Canon.” [8] Today, the concept of using simple electromagnetic properties to propel an object at high velocities remains the same. The difference may exist in whether or not technological advances could make a “usable” railgun a reality. Numerous countries, including the United States, continue to study the problems associated with the railgun, and all would probably agree that there are three key problems, which dominate the top of the list.

1. Rail Erosion

Similar to the problem faced with the “Super-gun” in the 1980’s, bore erosion or in this case, “rail” erosion, continues to be a concern. As a promising future weapon, an EM railgun would be of no value if the life expectancy of the barrel were equal to that of only a few shots. Therefore, one of the developmental challenges that engineers must face is rail survivability. For example, it may be of interest to investigate how high current densities behave and cause collateral damage to the barrel. Or, it may be of other interest to investigate what types of materials may be able to withstand friction and high current at the rail-projectile-rail interface. These are but a couple of examples, but none-the-less must still be overcome.

2. Fire-control and Guidance

Although not apparently obvious, guiding a projectile to its target is a significant challenge. As civilizations evolve, so does modern warfare tactics. The need for target accuracy is of high importance when it comes to minimizing civilian casualties. Therefore, if a naval EM gun is used from hundreds of miles away, a guidance system of some sort will have to be employed in order to assist the projectile to its target. However, a projectile accelerating out the barrel of an EM gun may undergo extreme g-forces and/or ionization shielding as it travels rapidly through the atmosphere at high velocities. Consequently, engineers may want to investigate any further advances in the survivability of microelectronics within a “high-g” environment. Or, another interest may be to study the employment of a satellite guidance system for “exo” then “endo”-atmospheric projectiles. These and other problems must be addressed.

3. Pulsed Power Supply

Historically, conventional weapon systems were integrated onboard a platform such that sufficient prime power or energy storage would be included in the platform to allow the weapon and vehicle propulsion to operate independently. [10] However, because of the high power requirements to “spark off” an EM railgun, providing an independent power supply may be very costly – monetarily and spatially. Engineering challenges include the design of a smaller high-density power source. Or, with the recent

advent of the “all electric” drive, engineers may have to develop an integration and/or exploitation scheme so that an EM gun could tap into the power supply normally used for propulsion. Regardless, research is still needed for pulsed power storage and switching systems to help bring a railgun into reality today. [3]

D. ELECTROMAGNETIC (EM) GUN THEORY

1. Electromagnetic Launch

The basis behind electromagnetic launch technology is the interaction between electrical current and magnetic fields. This interaction is known as the Lorentz Force and is defined by:

$$\vec{F} = q(\vec{v}_d \times \vec{B}) \quad (1.1)$$

As current passes through the rails, a magnetic field builds up between the rails according to the Biot-Savart law. Subsequently, as electrical current passes through the projectile/armature, the current drift velocity vector changes such that the magnetic field exerts a force upon any charged particles between the rails. Figure 1.D.1 illustrates this interaction.

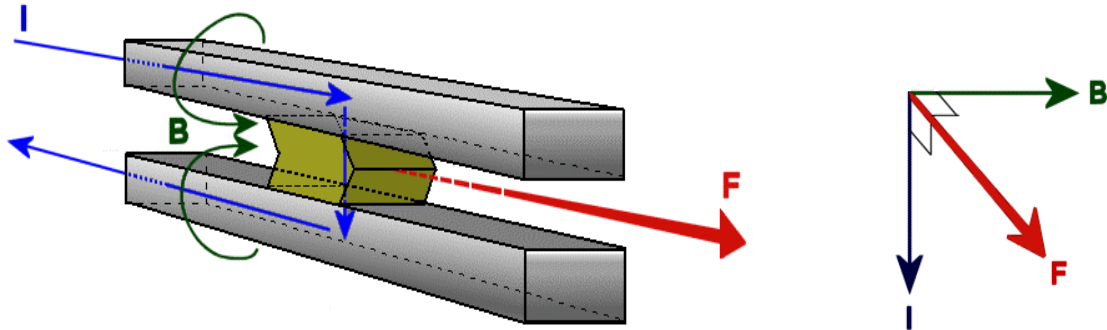


Figure 1.D.1 (left) Current and magnetic field interaction (right) Lorentz Force Law

However, to obtain a better understanding of the forces involved, we must first re-examine the Lorentz force. The magnitude of the Lorentz force can be written as:

$$\vec{F} = q(|\vec{v}_d| |\vec{B}|) \quad (1.2)$$

where q is an element of charge, v_d is the drift velocity of the charge, and B is the magnetic field created between the rails. As we continue to follow the path of the charge q over time we undergo a current I , which yields the following:

$$q = It = I \frac{\ell}{v_d} \quad (1.3)$$

where ℓ is the distance traveled by the charge q within the projectile. Figure 1.D.2 illustrates this relationship.

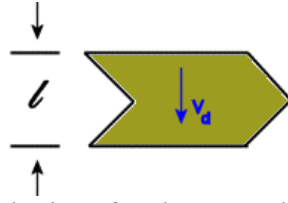


Figure 1.D.2 The drift velocity of a charge q along a projectile of height ℓ .

Substituting Equation 1.3 into Equation 1.2 and taking an infinitesimal step along the armature height ℓ we have:

$$d\vec{F} = dq(\vec{v}_d)\vec{B} = \left(I \frac{dx}{v_d} \right) (\vec{v}_d)\vec{B} = \vec{B}I dx \quad (1.4)$$

Note, that Equation 1.4 now shows a relationship between the magnetic field, the electrical current and the Lorentz force acting on the armature in accordance with the right hand rule. However, in order to specifically determine the function of the magnetic field B , we return to what we already know from the Biot-Savart Law. The magnetic field created from a current within a semi-infinite straight wire is:

$$\vec{B} = \frac{\mu_o I}{4\pi r} \quad (1.5)$$

where μ_o is permeability of free space, and r is the radial distance from the center of the wire. However, we must now make two assumptions: 1) the current passes through the center of the rails, and 2) the magnetic characteristics of the rectangular rails are similar to that of long round wires. Figure 1.D.3 illustrates these assumptions.

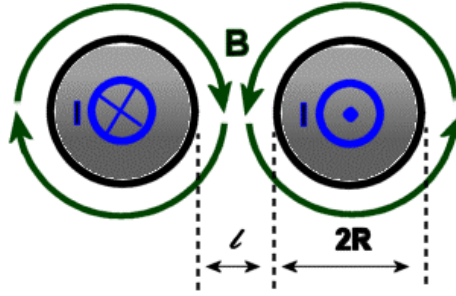


Figure 1.D.3 The magnetic field created by a current passing through the rails

Therefore, by substituting Equation 1.5 into 1.4 and integrating, the Lorentz force between the rails becomes approximately

$$F = \frac{\mu_o I^2}{4\pi} \int_R^{R+l} \left(\frac{1}{x} + \frac{1}{2R+l-x} \right) dx \quad (1.6)$$

After some integration and simplification we get:

$$F = \frac{\mu_o I^2}{4\pi} \ln \left\{ \frac{(R+l)^2}{R^2} \right\} \quad (1.7)$$

The significance of Equation 1.7 is the term for which we now define in Equation 1.8. L' is known as the inductance gradient, which has the units of (henries/meter). It is important to note that L' is not an inductance of the system. Instead, L' is a magnetic field factor, which is only dependent upon the geometry of the railgun itself. Therefore, L' remains constant once the railgun has been constructed.

$$L' \equiv \frac{\mu_o}{2\pi} \ln \left\{ \frac{(R+l)^2}{R^2} \right\} \quad (1.8)$$

Now, by substituting Equation 1.8 into 1.7, the magnitude of the Lorentz force can now be simply expressed as:

$$F = \frac{1}{2} L' I^2 \quad (1.9)$$

2. Circuit Structure

Apart from directly analyzing the Lorentz force, previous studies at the Naval Postgraduate School have recognized that the railgun system could also reasonably be modeled as an RLC circuit. John P. Hartke analyzed the model shown in Figure 1.D.4, and derived an expression for the force exerted by the railgun such that it would be comparable to the expression in Equation 1.9.

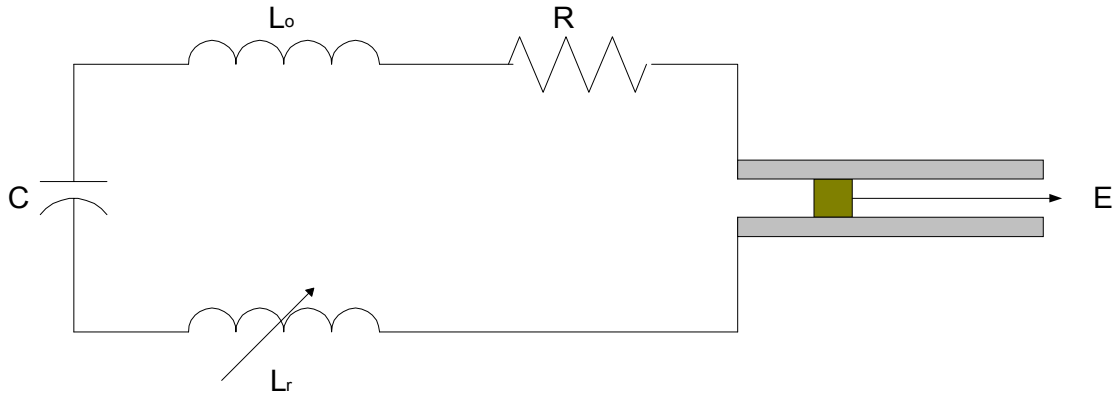


Figure 1.D.4 An ideal railgun circuit from Ref. [6]

Where C is the capacitive current source, L_o is the characteristic inductance of the system, R is the resistance of the system, and L_r is the variable inductance of the system as the projectile travels down the rails.

The details can be found in reference [6], but the concluding outcome in Equation 1.10 can be found from several equation transformations while adhering to the conservation of energy and Kirchhoff's Law. Subsequently, the force can be expressed as:

$$m \frac{dv}{dt} = \frac{1}{2} I^2 \frac{dL}{dx} \quad (1.10)$$

Where m is mass of the projectile, x is distance along the rails, and $\frac{dL}{dx}$ is comparable to L' in form and magnitude.

We can use this circuit model of the railgun to estimate the performance of a particular railgun. We can also use other parameters, such as acceleration, muzzle velocity and barrel length to model the design and performance of a future railgun. In this thesis, these parameters will be used to investigate a possible power supply for an existing railgun.

II. EM GUN OPTIMIZATION

A. EXISTING RAILGUN

The Naval Postgraduate School (NPS) Physics Department currently possesses a 1.2-meter electromagnetic railgun designed by Michael M. Lockwood. The 1.2-meter railgun as shown in Figure 2.A.1 allows for experimentation of various projectile sizes and rail configurations. [5]

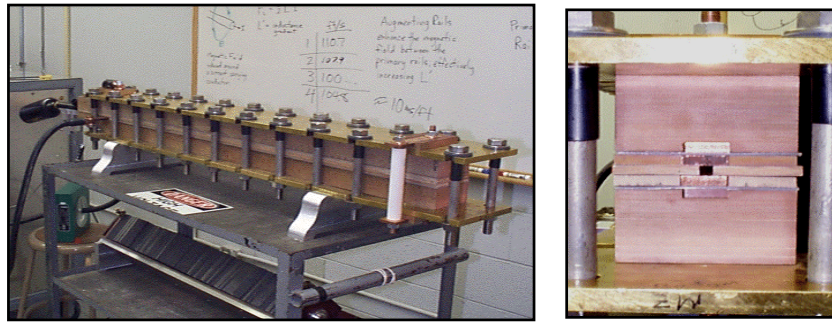


Figure 2.A.1 (left) NPS 1.2 meter railgun, (right) Muzzle view from Ref. [5]

1. Launcher Design

The current railgun design implements a series trans-augmentation configuration increasing the inductance gradient L' and subsequently increasing the magnitude of the Lorentz force. Figure 2.A.2 illustrates this augmentation.

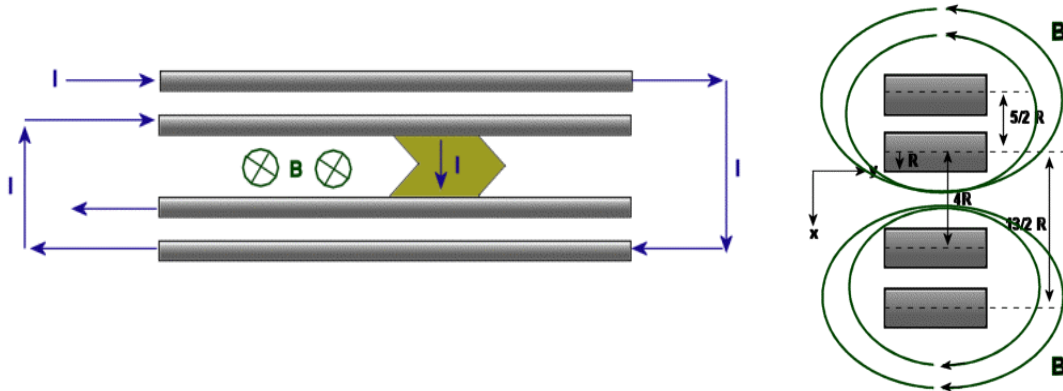


Figure 2.A.2 (left) Series trans-augmentation configuration (right) Magnetic field generated from a series trans-augmentation configuration after Ref. [5]

With the addition of the two outer rails, a greater magnetic field is generated as shown in Figure 2.A.2. As a result, the inductance gradient L' must be recalculated for this specific railgun geometry. Using a similar method to that used in Equation 1.7, the inductance gradient can be found from:

$$F = \frac{\mu_o I^2}{4\pi} \int_R^{3R} \left(\frac{1}{x} + \frac{1}{4R-x} + \frac{1}{\frac{5}{2}R-x} + \frac{1}{\frac{13}{2}R-x} \right) dx \quad (2.1)$$

Again, with a little integration and simplification, we find that the new inductance gradient is:

$$L' = \frac{\mu_o}{2\pi} \left\{ \ln 3 + \ln 3 + \ln \frac{11}{7} + \ln \frac{11}{7} \right\} = 6.2023E-7 \quad (2.2)$$

2. Power Unit Design

In addition to the railgun, Lockwood modified a previous power supply as the electrical current source of his gun. Figure 2.A.3 below displays the power supply.

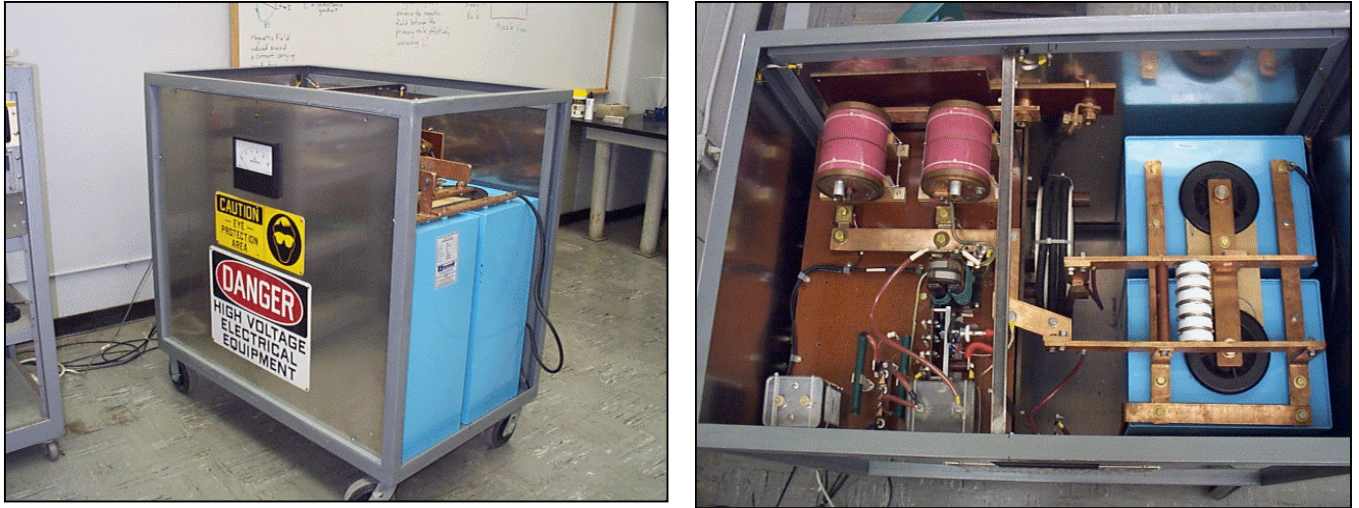


Figure 2.A.3 Naval Postgraduate School railgun power supply from Ref [5]

The power supply uses two Maxwell 830 μ f, 10kV high-energy capacitors, which provide up to 83kJ of energy. Main power switching between the power source and the rails is

achieved by using two TVS-40 vacuum switches, where each switch is capable of operating up to 20kV/100kA. To prevent current feedback or oscillation to the capacitors, the power unit configuration “crowbars” the capacitors after the current reaches its peak value and voltage on the capacitors start to reverse.[5] As a high-energy storage unit, the power supply has the potential to effectively fire the railgun at great muzzle velocity. However, in reference [5], it was pointed out that average muzzle velocity was only 30 m/s from the 1.2-m railgun. Hence, if we assume that the friction effects were small, compared to the high acceleration of the projectile, then the power supply was probably not sufficient. This thesis examines the capacitance required to fire the 1.2-meter railgun with a high muzzle velocity, e.g., 1 km/s.

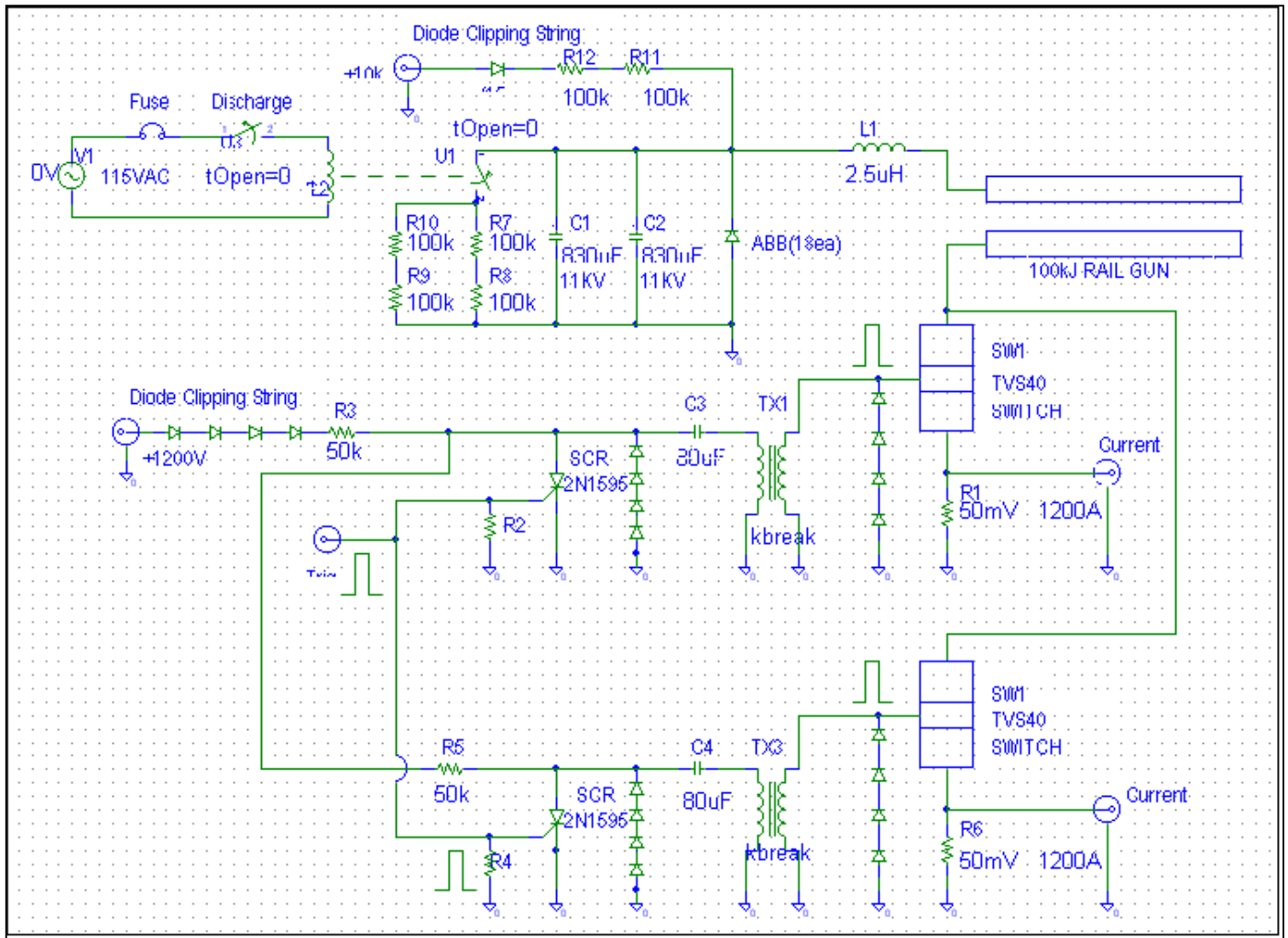


Figure 2.A.4 Power supply design from Ref. [5]

3. Projectile Design

Although numerous projectiles have been used at the Naval Postgraduate School, none were specifically designed for aerodynamic flight or lethality. Instead, the projectiles (Figure 2.A.5) were intended for initial railgun construction, as well as rail erosion analysis and experimentation. Nevertheless, each projectile was constructed to conduct the current between the rails and be subsequently propelled forward.



Figure 2.A.5 Lockwood's 1.2-meter railgun projectiles from Ref. [5]

For the purpose of this thesis, certain assumptions will be made regarding the size, shape, and mass of the projectile. We discuss the interactions of these parameters in rail gun performance.

B. A MULTI-VARIABLE PROBLEM

The purpose of this thesis is to design a conceptual power supply and projectile to match the 1.2-meter railgun system. The overall results depend on various trade offs. Several factors such as the muzzle velocity, rail length, projectile mass, and maximum current density are very much interrelated. For example, the amount of current applied to the railgun for a period of time determines the projectile acceleration and muzzle velocity. A shorter "applied current time" requires a larger current peak and vice-versa. Again however, the current peak determines the projectile mass because the projectile must have a minimum surface area in order to survive a maximum current density. Lastly, increased projectile mass affects the resultant acceleration. Hence, it is reasonable to assume that when it comes to building a railgun, a design must be found to

optimize or take advantage of each of these parameters. However, for the scope of this thesis, the railgun of interest has already been built. Therefore, we shall only investigate the prospect of providing a power supply, and possibly a subsequent projectile, for the existing railgun.

1. Assumptions

But, before proceeding it is important to note that some assumptions are made for the purpose of this thesis.

- a.) All effects of friction are neglected
- b.) All aerodynamic effects are neglected
- c.) The current passes from the rails to the projectile/armature homogenously throughout the surface area contact.
- d.) The projectile is solid and rectangular

2. Interdependence

As mentioned earlier, several factors depend on one another. Therefore, the following interdependencies are taken into consideration:

- a.) *Increasing Rail length increases acceleration time*
- b.) *Increasing Acceleration time decreases peak current*
- c.) *Increasing Current peak for the maximum current density increases projectile's contact surface area*
- d.) *Increasing Contact surface area increases projectile mass*
- e.) *Increasing Projectile mass decreases acceleration*
- f.) *Increasing Acceleration increases muzzle velocity*

3. Chosen Parameters for Rail Gun System Model

The following parameters are used for the existing railgun system:

Rail Length: 1.2 m	Muzzle Velocity: 1 km/s	Rail Separation: 0.00625 m
Inductance: 2.50E-6 H/m	Resistance: 0.003 Ω	L' : 0.6202E-6 H/m
Voltage: 10 kV	Projectile Density: 13.4 g/cm ²	

Table 2.B.1 Parameters for a Desired Naval Postgraduate School Rail Gun

C. CONSTANT CURRENT

Consider the case of constant current. That is to say, at time $t = 0$ the current would “turn on” with some value I_o , then immediately “turn off” once the projectile has left the rails at some time t . Figure 2.C.1 illustrates this ideal situation.

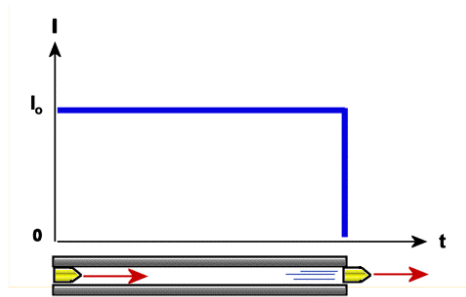


Figure 2.C.1 An ideal constant current for a railgun

Therefore, suppose we build a hypothetical power supply such that it would deliver enough charge to generate a constant current. If so, then from Equation 1.9, the constant current would provide a constant acceleration expressed by:

$$a = \frac{1}{2m} L' I_o^2 = \frac{1}{2m} L' \left(\frac{Q}{t} \right)^2 \quad (2.3)$$

Where Q is the amount of charge delivered in period of time t .

Subsequently, in order to find the velocity of the projectile we could integrate Equation 2.3 from time $t=0$ to $t=t$. However, since we are assuming a constant acceleration a , we can use what we know from simple one-dimensional motion. Therefore, the velocity of the projectile can be expressed as:

$$v = v_o + \frac{L'}{2m} \left(\frac{Q}{t} \right)^2 t = \frac{L'}{2m} I_o Q \quad (2.4)$$

Where V_o is the initial voltage of the hypothetical power supply, and the initial velocity $v_o = 0$. Correspondingly, the displacement of the projectile would then be:

$$x = x_o + \frac{1}{4m} L' \left(\frac{Q}{t} \right)^2 t^2 = \frac{1}{4m} L' Q^2 \quad (2.5)$$

With the initial displacement $x_o = 0$.

Now before proceeding, we must note that although we have simplified the expression for the projectile displacement as function of charge or capacitance, the expression for the projectile velocity remains as function of charge and peak current. To get a feel for dependence on C and L , take the constant current I_o to be the peak current from a single capacitor, and $Q = CV_o$. Calculation of I_o is discussed later in this thesis to

be $I_o = V_o \sqrt{\frac{C}{L}}$. Substituting this into Equation 2.4, we now get:

$$v = \frac{L'}{2m} I_o (C V_o) = \frac{L'}{2m} V_o^2 C \sqrt{\frac{C}{L}} \quad (2.6)$$

Where V_o is the initial voltage on the capacitor.

We can use Equations 2.5 and 2.6 to determine the hypothetical amount of capacitance required for a “constant current” power supply. In order to not make this case more complicated, we choose an arbitrary projectile mass of 0.222 kg, which we can later compare to another model in this thesis. We find the following results apply for a 0.222-kg projectile and the conditions in Table 2.B.1:

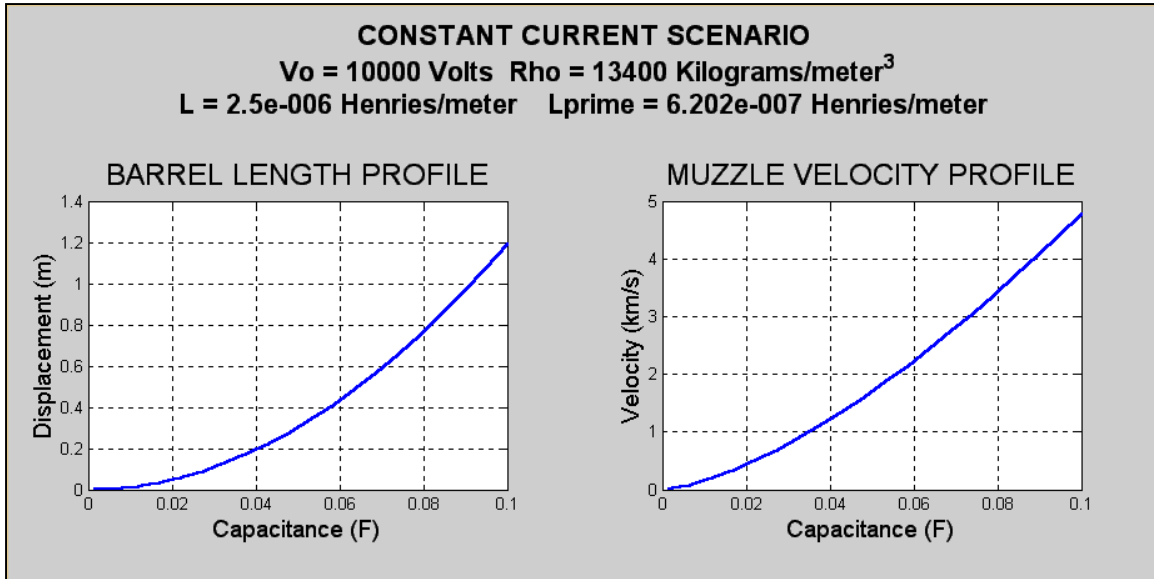


Figure 2.C.2 Railgun performances for a constant current

Figure 2.C.2 shows that in order for the projectile to accelerate 1.2 meters under a constant current, we would require at least 100 mF of capacitance. In addition, for the same amount of capacitance, we would well exceed the desired velocity of 1 km/s. Although 100 mF would seem rather large, as compared to a real-life scenario, this would agree with the notion of providing a so-called “capacitive constant current” source. That is to say, we would require an enormous amount “charge” delivered in order to simulate a steady current. Nevertheless, the constant current scenario gives us a simple benchmark

to compare with later calculations. Now we analyze the characteristic acceleration, velocity and displacement of a railgun system, for a one-meter railgun and power supply like that in Figure 1.D.4.

D. PULSE POWER

In reality, although our existing power supply can store high energy densities and maintain high voltages, it does not provide a constant source of current to the rail gun.

Instead it generates a time dependent current pulse, which may vary in magnitude and width depending on the voltage, capacitance and inductance of the system. These same factors can be chosen so as to modify the shape of the current pulse.

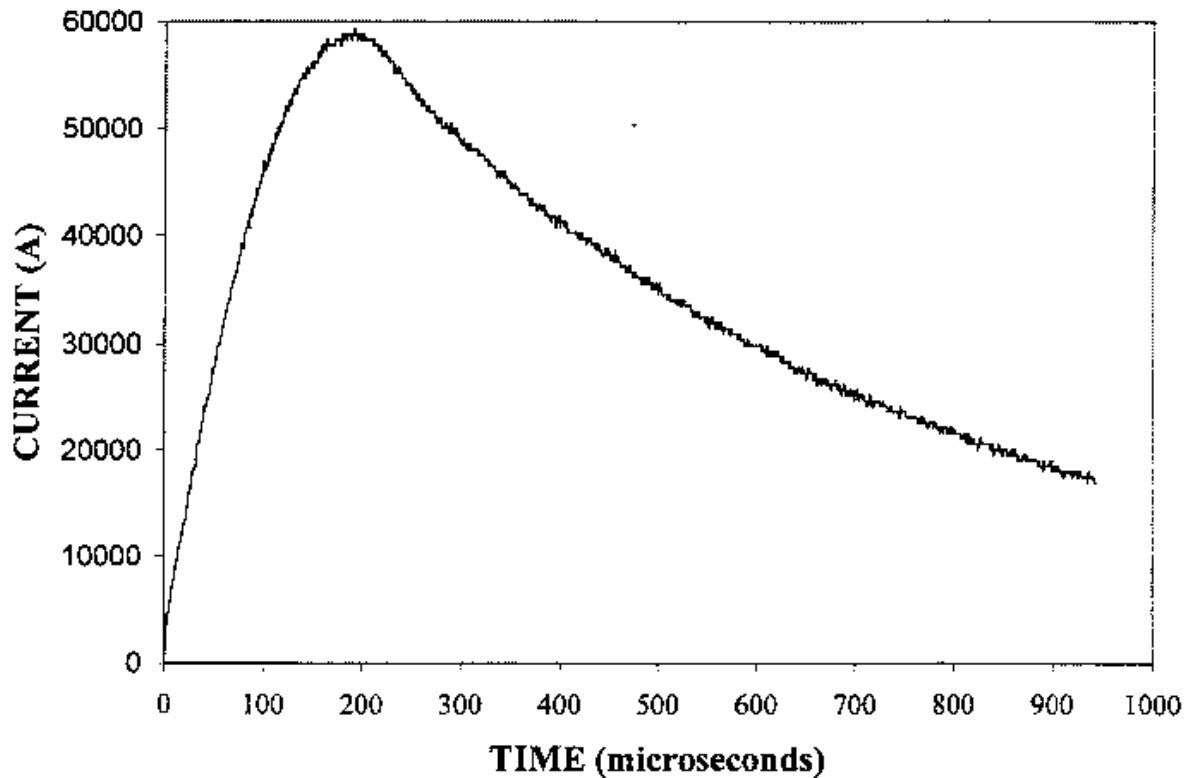


Figure 2.D.1 A single current pulse from Lockwood's railgun power supply[5]

Examining Figure 2.D.1, shows that the current pulse appears to rise sinusoidal until it reaches its peak I_o at some time t' , then falls off exponentially at infinity. Thus, we can reasonably model the rise in current as:

$$I = I_o \sin(\omega t) \quad (2.7)$$

Correspondingly, the fall off in current can be modeled as:

$$I = I_o e^{-\frac{R}{L}t} \quad (2.8)$$

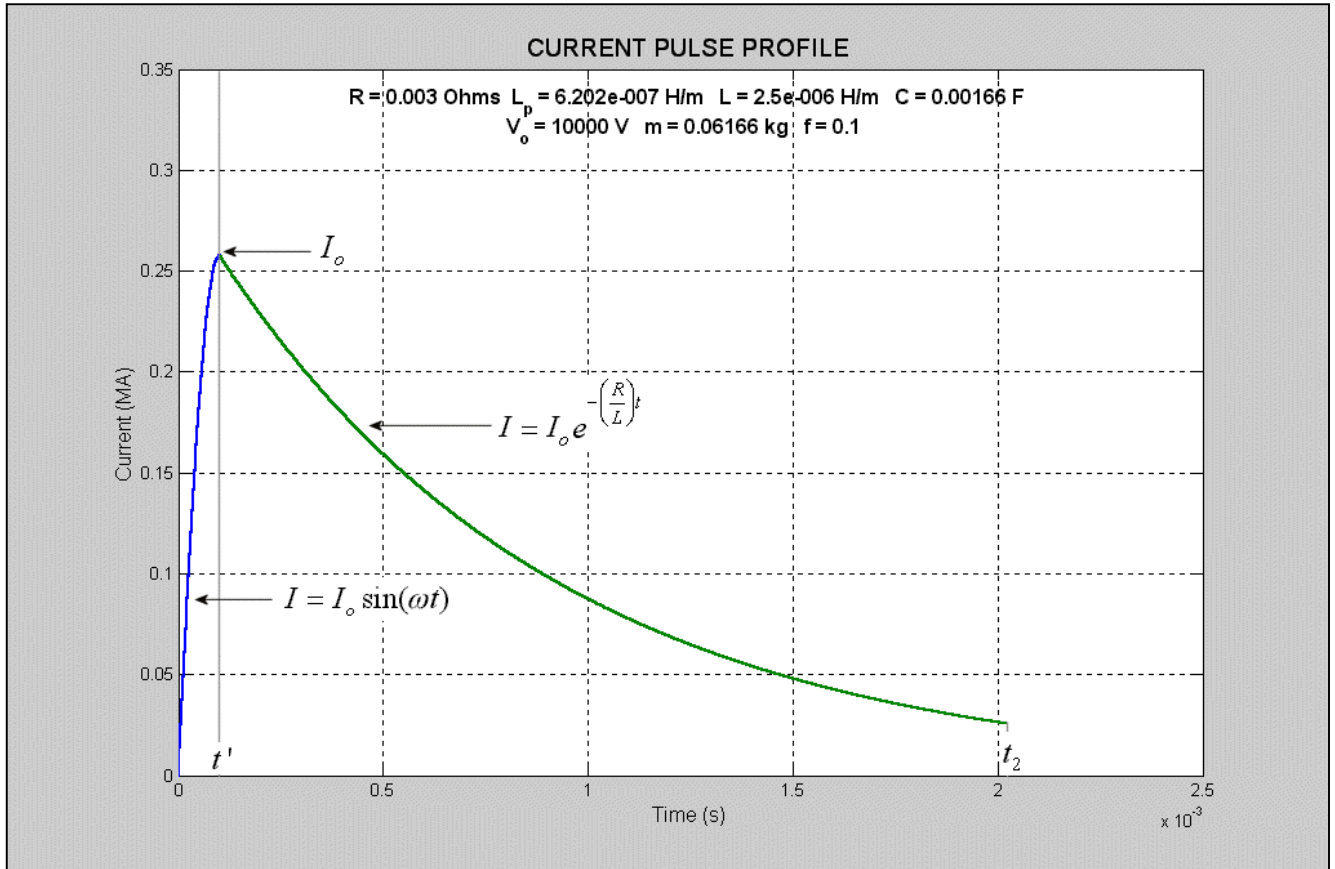


Figure 2.D.2 MATLAB current pulse model

1. Capacitor Energy Transfer

Assuming the railgun system acts as a perfect LCR oscillator, the energy is transferred from the capacitor when time $t = 0$ to $t = t'$. Therefore, we need to determine when t' occurs. Beginning with Equation 2.7, the current I reaches its peak I_o when

$$\sin(\omega t) = 1 \quad (2.9)$$

where $\omega = \frac{1}{\sqrt{LC}}$. Substituting ω into Equation 2.9 and solving for t we get:

$$t = t' = \frac{\pi \sqrt{LC}}{2} \quad (2.10)$$

where again, t' is the time at which the capacitor has discharged completely.

a. Projectile Acceleration Due to Rise in Current

Now by examining Equation 1.9, it is simple to observe the relationship between the acceleration of the projectile and the shape of the current pulse. The acceleration due to the capacitor discharge is directly proportional to

$$I^2 = I_o^2 \sin^2(\omega t) \quad (2.11)$$

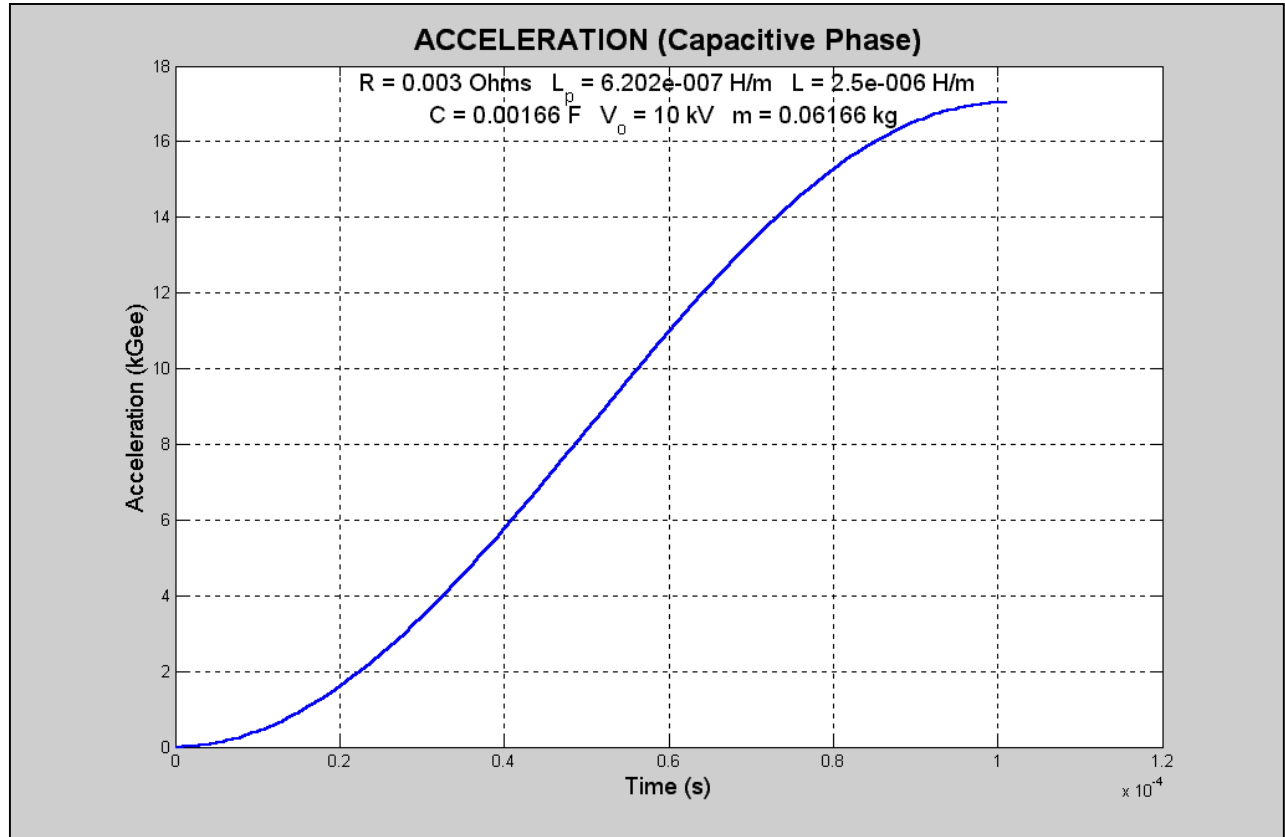


Figure 2.D.3 Acceleration due to capacitor discharge

b. Projectile Velocity Due to Rise in Current

The velocity from the capacitive discharge can then be determined by substituting Equation 2.11 into 1.9 and integrating from time 0 to t' . Consequently, we get:

$$v_1 = \frac{1}{2m} L' I_o^2 \int_0^{t'} \sin^2(\omega t) dt \quad (2.12)$$

Hence,

$$v_1 = v_o + \frac{1}{2m} L' I_o^2 \left(\frac{t}{2} - \frac{\sin(2\omega t)}{4\omega} \right) \Bigg|_0^{t'} \quad (2.13)$$

From Equation 2.10, we can substitute for t' and find:

$$v_1 = v_o + \frac{1}{4m} L' I_o^2 \left(\frac{\pi \sqrt{LC}}{2} \right) \quad (2.14)$$

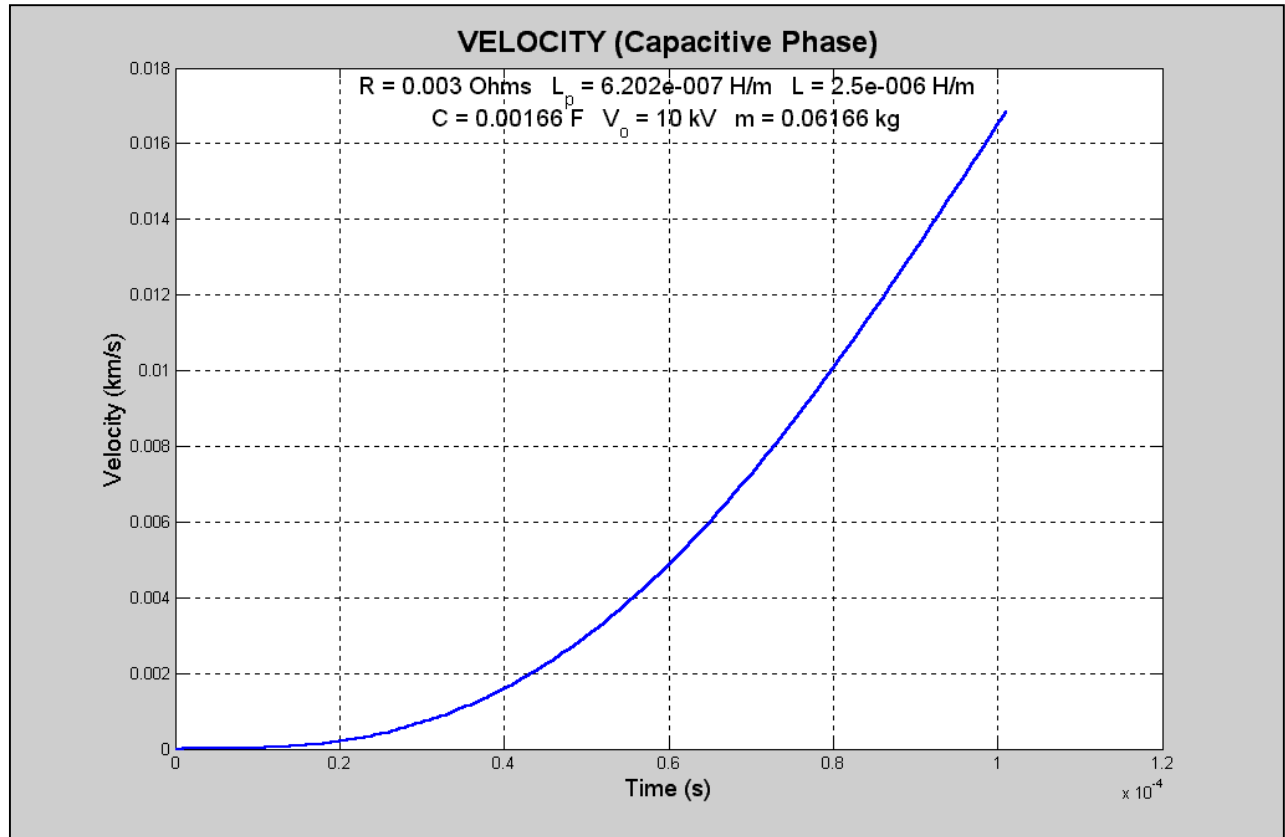


Figure 2.D.4 Velocity due to capacitor discharge

c. Projectile Displacement Due to Current Rise

We can determine the displacement of the projectile within the rails from by solving Equation 2.12 as a function of t , then integrate from 0 to t' . Thus, we get:

$$x_1 = \int_0^{t'} v_1(t) dt = \int_0^{t'} v_o + \frac{1}{2m} L' I_o^2 \left(\frac{t}{2} - \frac{\sin(2\omega t)}{4\omega} \right) dt \quad (2.15)$$

Hence,

$$x_1 = x_o + v_o t + \frac{1}{2m} L' I_o^2 \left[\frac{t^2}{4} + \left(\frac{\cos(2\omega t)}{8\omega^2} \right) \right] \Bigg|_0^{t'} \quad (2.16)$$

Again substituting for t' and simplifying, we get:

$$x_1 = v_o \left(\frac{\pi \sqrt{LC}}{2} \right) + \frac{L' I_o^2}{4m} \left(\frac{\pi^2 - 4}{8} \right) LC = v_o \left(\frac{\pi \sqrt{LC}}{2} \right) + \frac{L' I_o^2}{4m} (0.7335) LC \quad (2.17)$$

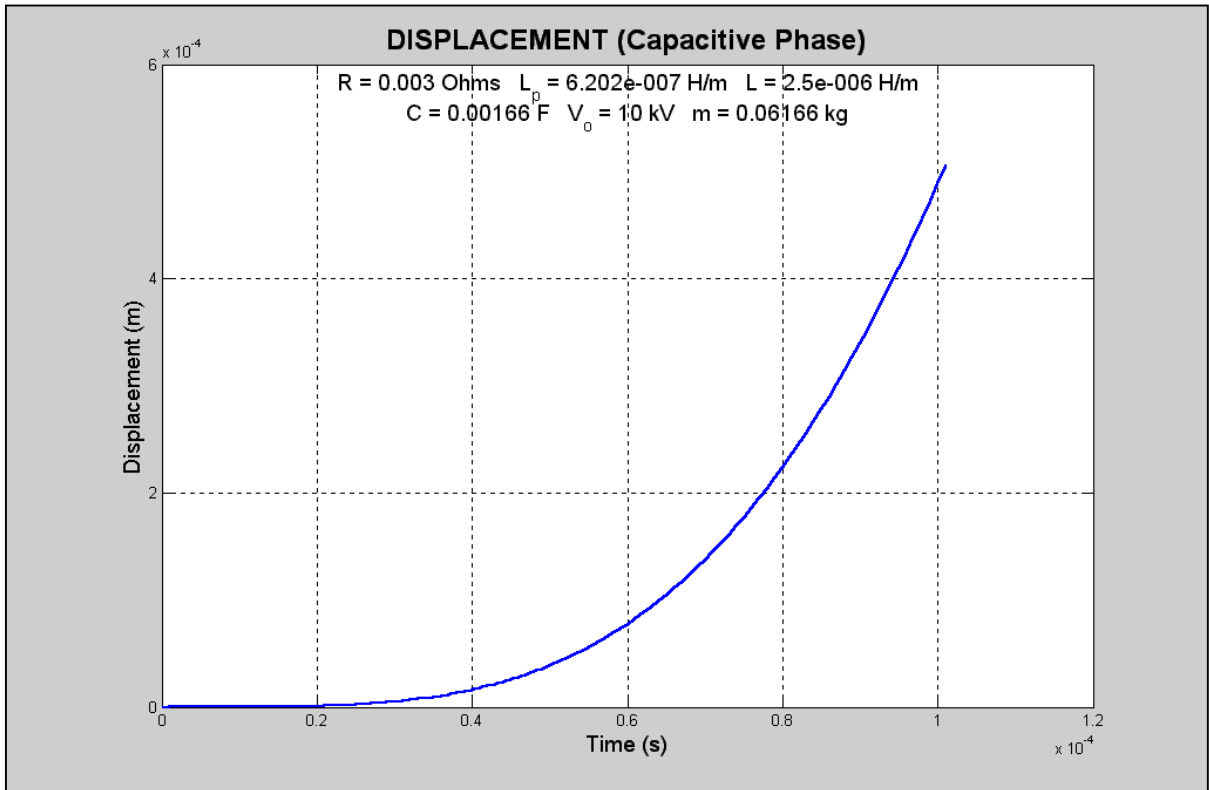


Figure 2.D.5 Projectile displacement due to capacitor discharge

2. Inductive Transfer Phase

Looking back at Figure 2.D.1, although the inductive energy falls off to zero at infinity, the rails are finite and must sustain some “left over” energy when the projectile exits the barrel. However, it may be possible to choose a railgun length such that we are left with a desired fraction f of the peak current remaining at the end of the barrel. In order to begin, we must first determine the time t_2 at which the desired “cutoff” takes place. Examining Equation 2.8, the “cutoff” occurs when we are left with some fraction f of I_o . That is to say, we have the following:

$$fI_o = I_o e^{-\frac{R}{L}t} \quad (2.18)$$

Solving for t we get:

$$t = t_2 = (t_f - t') = -\frac{L}{R} \ln(f) \quad (2.19)$$

In addition, it should be noted, that after the capacitor has completely discharged, the time t_2 is equivalent to the remaining time $(t_f - t')$.

a. Projectile Acceleration Due to Inductance

From our current pulse model in Figure 2.D.1, the current falls off exponentially. Therefore, we should also expect the acceleration to fall off exponentially. This is because the acceleration fall off is directly proportional to:

$$I^2 = I_o^2 e^{-2\frac{R}{L}t} = f^2 I_o^2 \quad (2.20)$$

Again however, since we have chosen to leave a fraction f of the peak current at the end of the rails, the acceleration fall off will be bounded by the same time frame as that of the fall off current. This is to say the acceleration would only occur from time $t = t'$ to $t = t_2$.

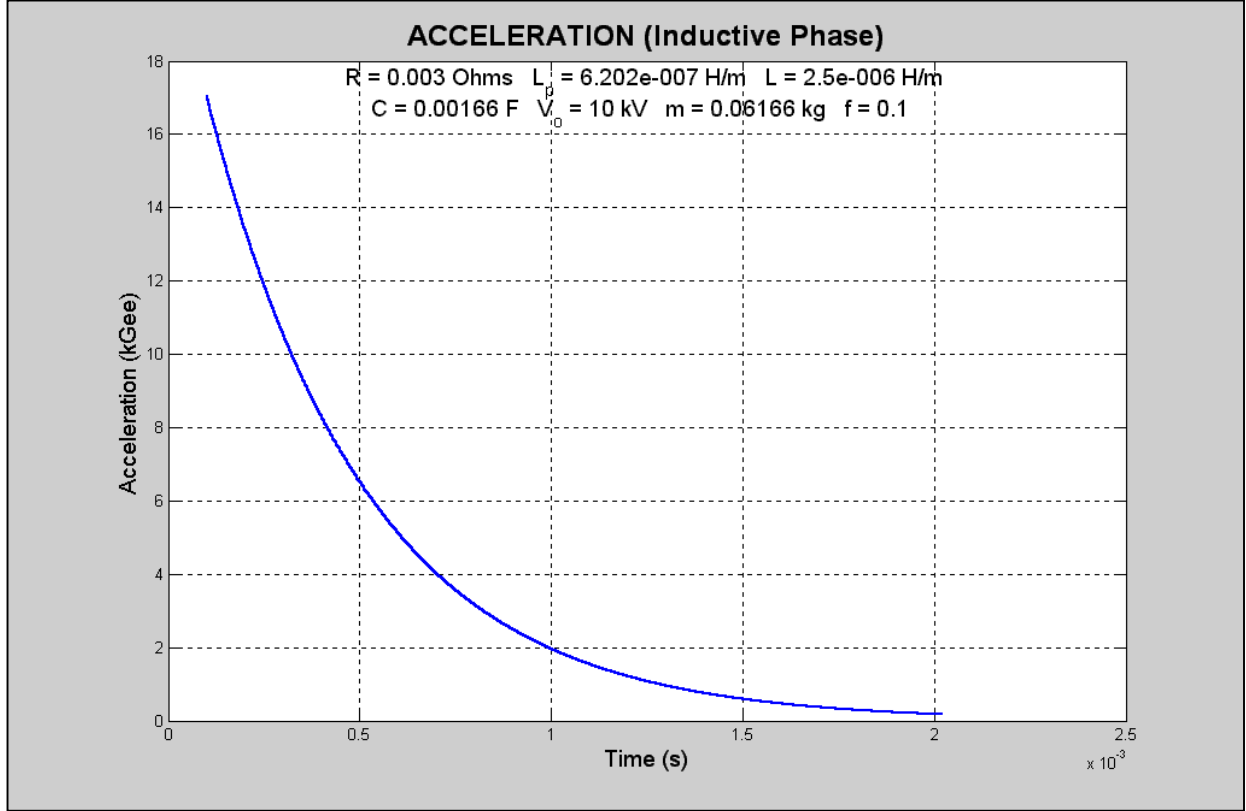


Figure 2.D.6 Acceleration due to inductance versus time. $f = 0.1$ at $t = 2$ ms.

b. Muzzle Velocity

By using a similar integration technique as that used to determine the velocity produced by the capacitive discharge, we can determine the increase in velocity during the inductive phase. However, since we are interested in the final velocity or muzzle velocity of the projectile at some time t_2 , we cannot simply consider the inductive curve only from time t' to t_2 . This is because the final velocity includes the initial velocity due the capacitive discharge at time t' . Therefore, we begin with:

$$v_f = v_1 + \frac{1}{2m} L' I_o^2 \int_{t'}^{t_f} e^{-\frac{2R}{L}(t-t')} dt \quad (2.21)$$

where v_1 is the initial velocity due to the capacitive discharge. So,

$$v_f = v_1 + \frac{1}{2m} L' I_o^2 \frac{-L}{2R} \left(e^{-\frac{2R}{L}(t-t')} \right) \bigg|_{t'}^{t_f} \quad (2.22)$$

Substituting Equation 2.19 for $(t_f - t')$ and simplifying, we get:

$$v_f = v_i + \frac{1}{4m} L' I_o^2 \frac{L}{R} (1 - f^2) \quad (2.23)$$

Again substituting Equation 2.14 for v_i and simplifying, we now have:

$$v_f = v_o + \frac{L' I_o^2}{4m} \left(\frac{\pi \sqrt{LC}}{2} + \frac{L}{R} (1 - f^2) \right) \quad (2.24)$$

Figure 2.D.7 illustrates the velocity profile as the current falls off due to some inductance in the system. Note how the velocity begins to level off as the current falls off.

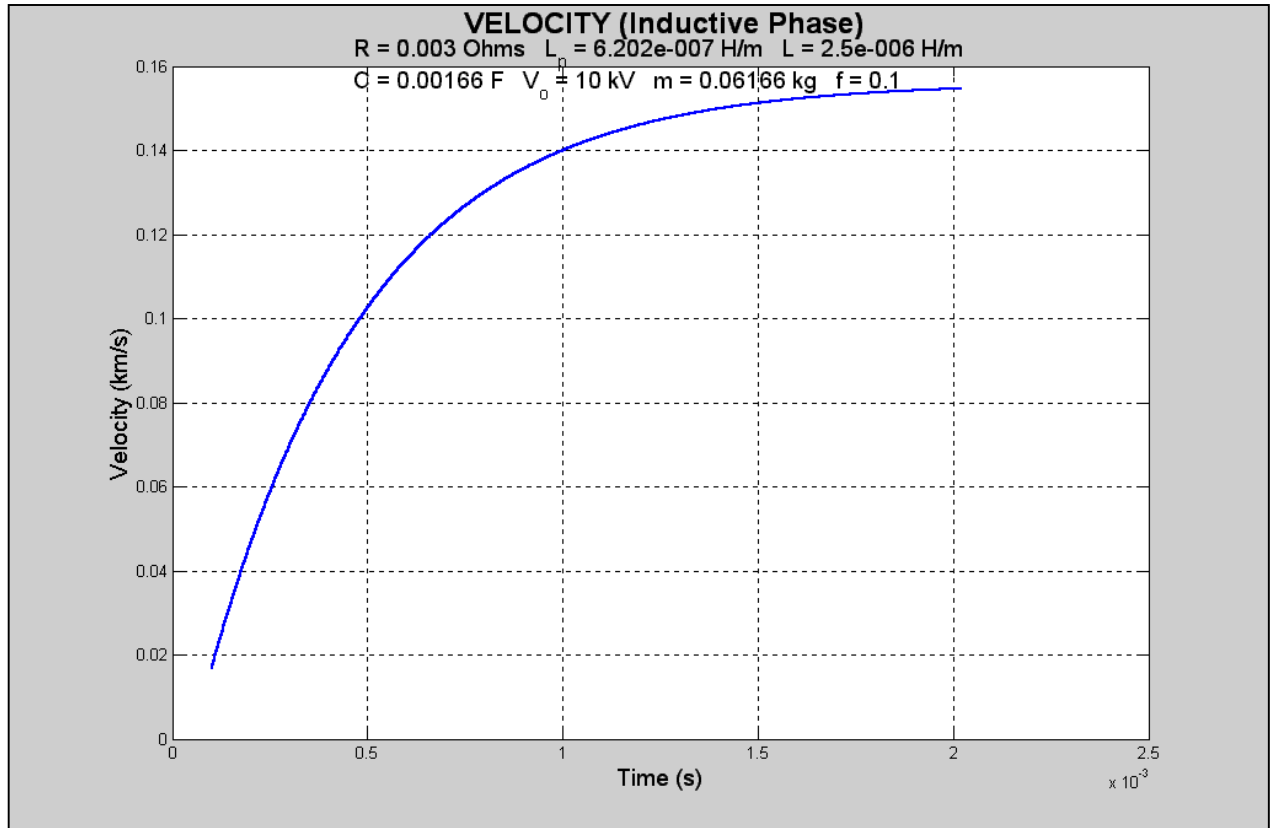


Figure 2.D.7 Projectile velocity due to inductance versus time. $f = 0.1$ at $t = 2$ ms.

c. *Final Projectile Displacement or Barrel Length*

We can now determine how long the rails must be in order to carry a projectile with the Lorentz force from start to finish. Again, the railgun length must take

into account the desired fraction of the current peak left at the end of the rail. By including the initial displacement traveled by the projectile due to the capacitive discharge up to time $t = t'$, we have:

$$x_f = x_1 + \int_{t'}^{t_f} v_f(t) dt \quad (2.25)$$

By solving Equation 2.21 as function of t instead of t_f , we can substitute the result into Equation 2.25 to give:

$$x_f = x_1 + \int_{t'}^{t_f} v_1 dt + \int_{t'}^{t_f} \frac{1}{4m} L' I_o^2 \frac{L}{R} \left[1 - e^{-2\frac{R}{L}(t-t')} \right] dt \quad (2.26)$$

So,

$$x_f = x_1 + v_1 (t_f - t') + \frac{L' I_o^2}{4m} \frac{L}{R} \left\{ t \Big|_{t'}^{t_f} - \left(-\frac{L}{2R} \right) e^{-2\frac{R}{L}(t-t')} \Big|_{t'}^{t_f} \right\} \quad (2.27)$$

Again, substituting Equation 2.19 for $(t_f - t')$ and simplifying, we get:

$$x_f = x_1 + v_1 \left(-\frac{L}{R} \ln(f) \right) + \frac{L' I_o^2}{4m} \frac{L}{R} \left\{ \left(-\frac{L}{R} \ln(f) \right) - \left(\frac{L}{2R} \right) [1 - f^2] \right\} \quad (2.28)$$

Finally, substituting Equation 2.17 for x_1 and simplifying, we now have:

$$\begin{aligned} x_f = v_o \left(\frac{\pi \sqrt{LC}}{2} - \frac{L}{R} \ln(f) \right) + \frac{L' I_o^2}{4m} \left\{ (0.7335) LC + \left[\frac{\pi \sqrt{LC}}{2} \left(\frac{-L}{R} \ln(f) \right) \right] \right\} + \dots \\ \dots + \frac{L' I_o^2}{4m} \left\{ \left(\frac{L}{R} \right)^2 \left[-\ln(f) - \left(\frac{1-f^2}{2} \right) \right] \right\} \end{aligned} \quad (2.29)$$

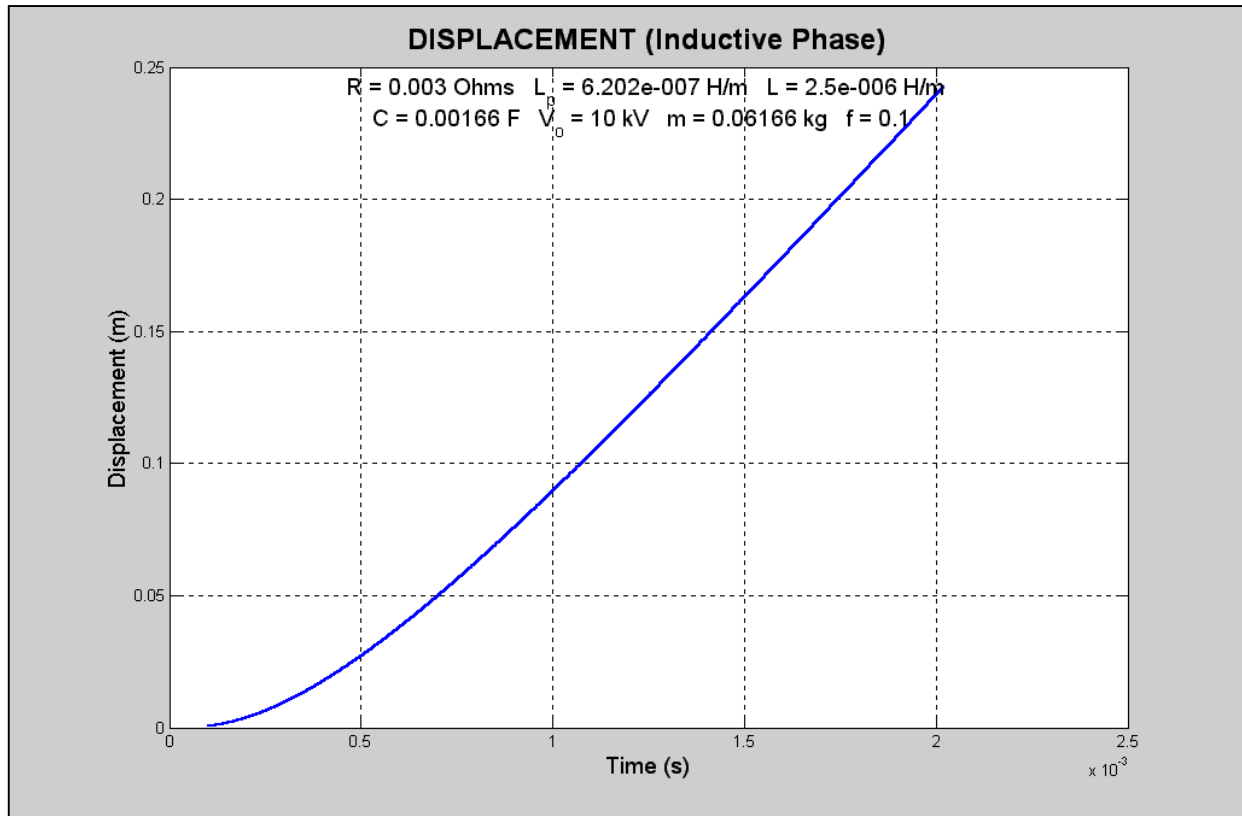


Figure 2.D.8 Projectile Displacement due to the Lorentz force versus time, from Equation 2.25. $f = 0.1$ at $t = 2 \text{ ms}$.

E. MASS AND CHARGE

Now that we have a working model derived from our current pulse, the only task that remains is to find the required amount charge that needs to be transferred through the projectile. In other words, we need to determine how much capacitance we need to add to our existing power supply. However, although we could guess at the amount of required capacitance by trial and error, because of the “interdependencies” we must take into consideration, a more general approach would probably be more prudent. The amount of acceleration is inversely proportional to the mass of the projectile. As a result, more capacitance will be required to accelerate a more massive projectile. Therefore, a generalized look at the barrel length and muzzle velocity as function of mass and capacitance would probably serve more useful.

1. Peak Current

Since, we can easily choose a range of capacitances, the projectile mass is the variable we are left with. In order to determine the mass, we must first calculate the expected peak current I_o . The amount of charge Q that is transferred from a generated current is

$$Q = \int Idt \quad (2.30)$$

Hence, as the capacitor discharges, the transferred charge due to the rise in current is:

$$Q_c = \int_0^{t'} I_o \sin(\omega t) dt \quad (2.31)$$

Now, integrating, and using Equation 2.10 for t' , we get:

$$Q_c = \frac{I_o}{\omega} \left[\left(-\cos\left(\frac{1}{\sqrt{LC}} \frac{\pi \sqrt{LC}}{2}\right) \right) - (-1) \right] = \frac{I_o}{\omega} \quad (2.32)$$

However, we know from the definition of capacitance that:

$$Q_c = CV_o \quad (2.33)$$

By substituting Equation 2.32 into 2.33 we now have:

$$\frac{I_o}{\omega} = CV_o \quad (2.34)$$

Therefore the peak current can be expressed as

$$I_o = \frac{CV_o}{\sqrt{LC}} = \frac{V_o \sqrt{LC}}{L} = V_o \sqrt{\frac{C}{L}} \quad (2.35)$$

2. Projectile Surface Area

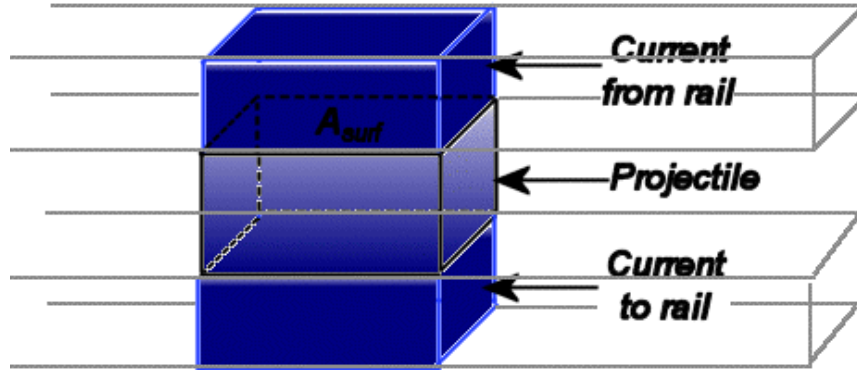


Figure 2.E.1 Homogeneous current density

Looking back to Chapter 2.B.1, we assumed that the current would pass homogeneously throughout the contact surface area of the projectile and rails (Figure 2.E.1). If so, the projectile would encounter a uniform current density. Now, by associating the projectile's "survivability" to the projectile's ability to withstand a high current density, a minimum contact surface area can be calculated for a given maximum current density. Equation 2.36 shows this calculation.

$$A_{surf} = \frac{I_o}{J_{max}} \quad (2.36)$$

Where A_{surf} is the minimum contact surface area, I_o is the peak current, and J_{max} is the maximum current density.

3. Projectile Mass

Subsequently, we can now calculate the projectile mass using the following basic steps:

- 1.) Calculate the peak current I_o

- 2.) Use I_o and the maximum current density J_{max} , calculate the minimum contact surface area A_{surf} of the projectile
- 3.) Use the surface area and the separation between the rails to calculate the projectile's volume

$$Volume_{proj} = A_{surf} \times separation_{rail} \quad (2.37)$$

- 4.) Finally, calculate the projectile's mass by using the volume and a desired material density ρ

$$mass = \rho \times Volume_{proj} \quad (2.38)$$

In summary, with an increase in capacitance, we should expect a larger surface area imparted to the projectile to keep the current density $J \leq J_{max}$. Of course, with the increase in surface area and subsequent volume, we would also expect an increase in the projectile mass. Figure 2.E.2 illustrates this simple concept.

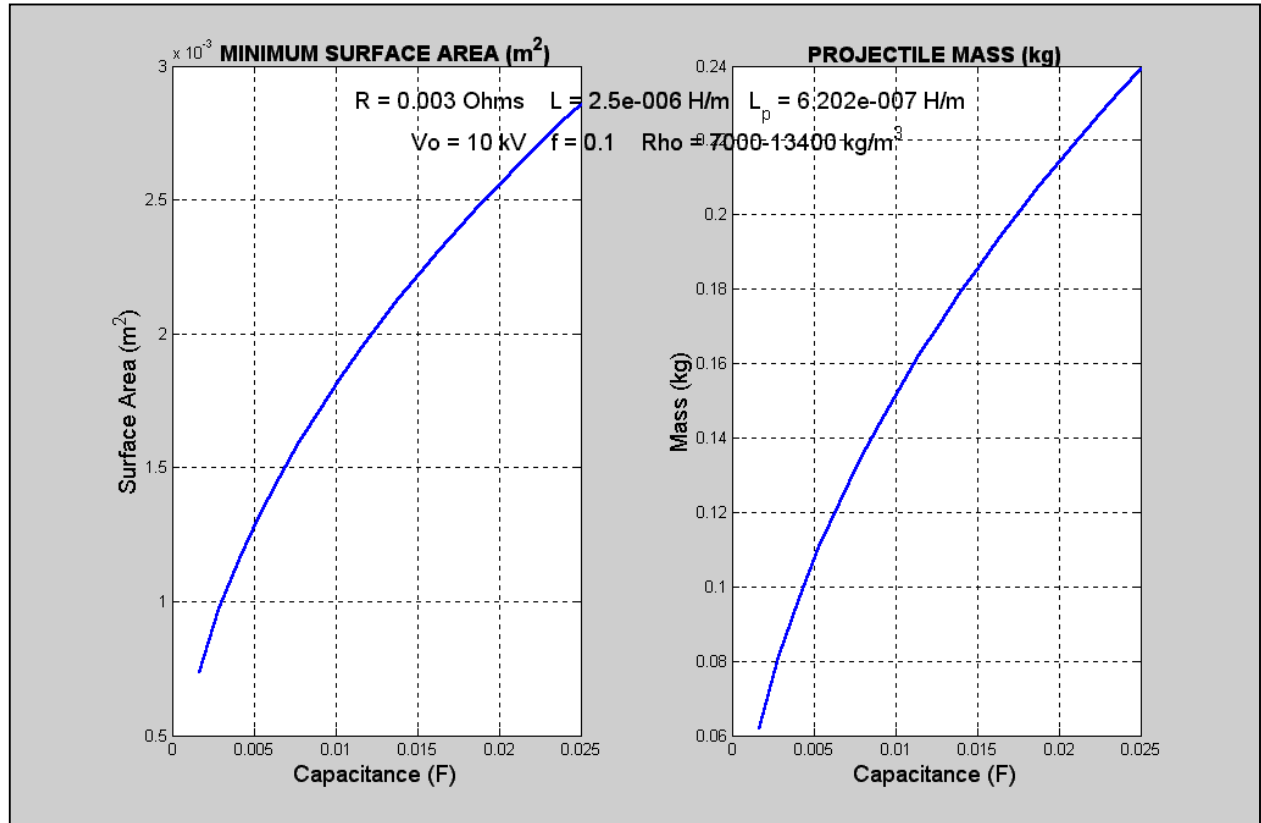


Figure 2.E.2 Area and mass versus capacitance

The mass also depends on the density of the projectile. We can manipulate both the volume and materials for which we intend to use to achieve a desired projectile mass.

F. BARREL LENGTH

We can now calculate a probable barrel length as function of capacitance and mass. However, for the purpose of this thesis, the barrel length has already been chosen to be 1.2 meters. Yet, it is important to note that the calculations made by Equation 2.28 actually tell us how far the Lorentz force should carry the projectile down the barrel for a given f . Therefore, we want have capacitance and mass that permit effective use of the entire length of the barrel. Figure 2.F.1 shows these calculations.

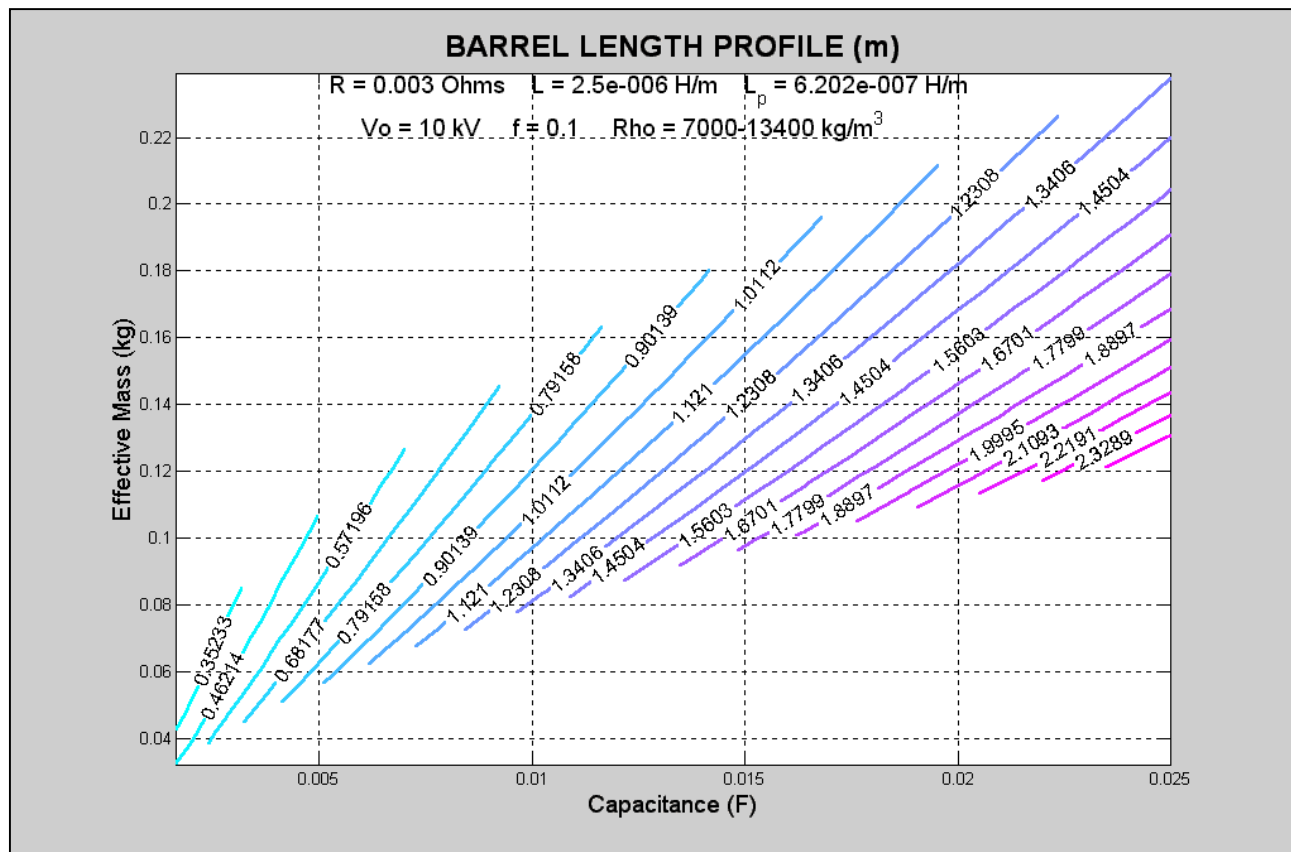


Figure 2.F.1 Barrel lengths as a function of capacitance and mass

Analyzing Figure 2.F.1 shows that there is a definite range of capacitance for which we can use, depending on the mass of the projectile. For a projectile mass of about 0.222 kg, we would need approximately 21.5 mF to carry the projectile for 1.2 m. However, if we used a material such that the density provided a projectile mass of only 0.145 kg, we would only need approximately 15.0 mF to carry the projectile 1.2 m. A smaller projectile mass would require a smaller capacitance.

G. MUZZLE VELOCITY

The next step would be to look at the respective muzzle velocities for these same capacitances and masses as those used in the previous section with $f = 0.100$. Therefore, the following muzzle velocities are calculated with Equation 2.24.

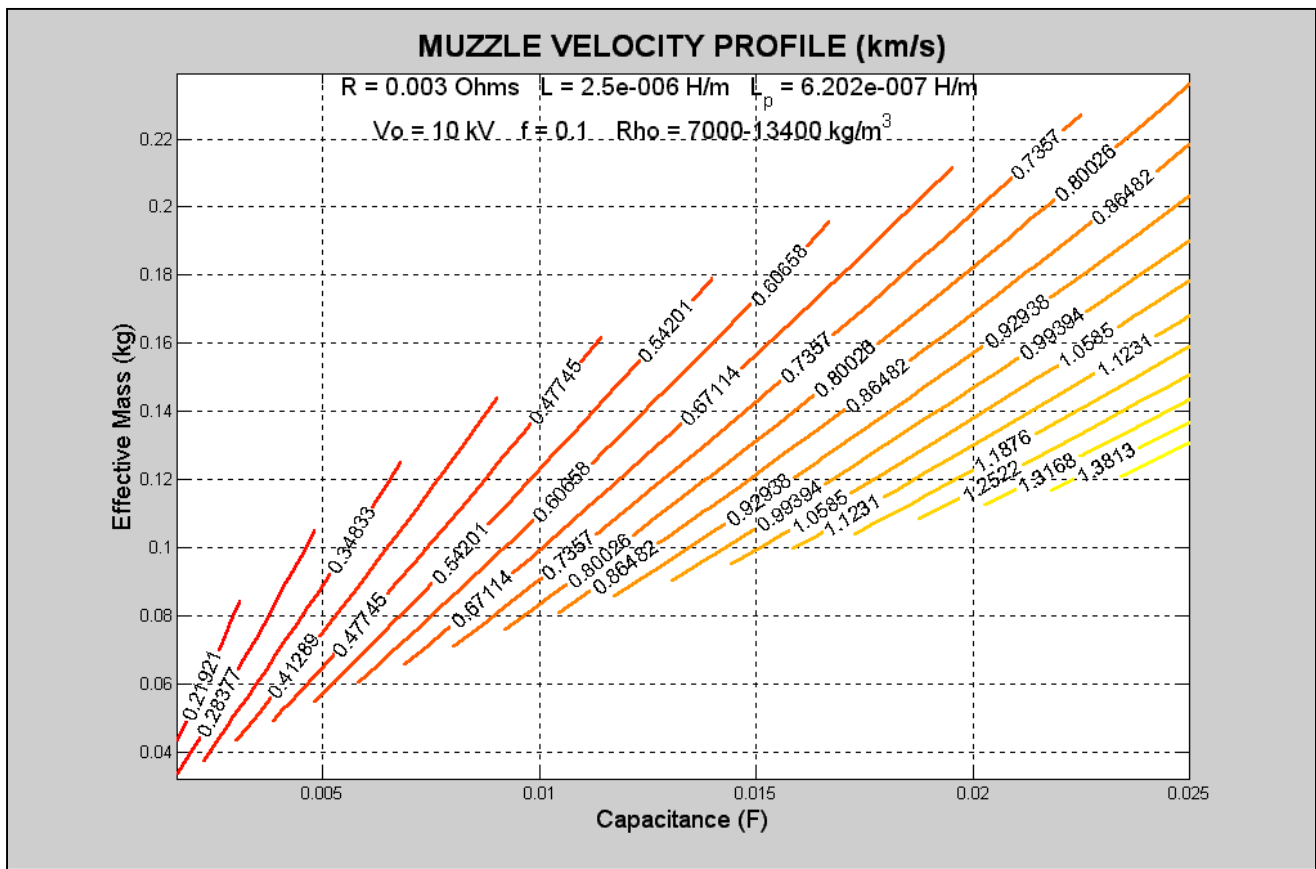


Figure 2.G.1 Muzzle velocities as a function of mass and capacitance

Analyzing Figure 2.G.1 shows that using 21.5 mF with a 0.222-kg projectile would only produce a 0.735 km/s muzzle velocity, which is under the specified goal of 1 km/s. Furthermore, if again you were to use a less dense material such that you had a 0.145-kg projectile and used 15.0 mF, you would still achieve the same 0.735 km/s muzzle velocity. Consequently, in order to increase the muzzle velocity we must do one of two things: 1) Increase the amount of capacitance or 2) Decrease the mass of the projectile.

1. Implication of Increasing Capacitance

A capacitance can be chosen such that the muzzle velocity of the projectile will achieve at least 1 km/s. For example, if we were to go from 21.5 mF to 36.5 mF, the muzzle velocity would increase from 0.735 km/s to 1.05 km/s. Figure 2.G.2 below illustrates this concept.

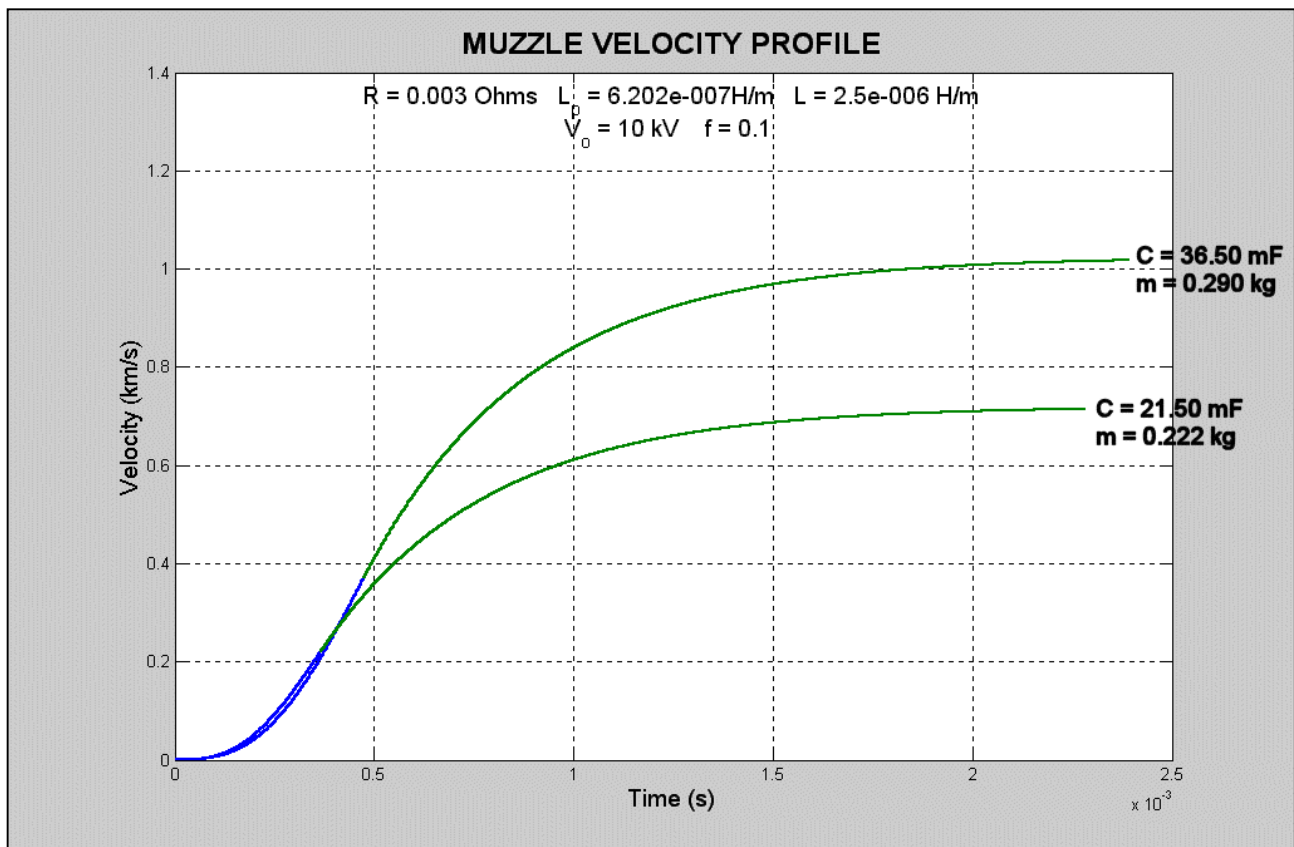


Figure 2.G.2 A comparison of muzzle velocity with increased capacitance. With 21.5 mF, $f = 0.1$ at $t = 2.3 \text{ ms}$, and with 36.5 mF, $f = 0.1$ at $t = 2.4 \text{ ms}$.

Now, we know from Equation 2.35 that an increase in capacitance demands an increase the current peak and a subsequent increase in mass. However, the change in mass may not be as significant as another disparity. Bear in mind that all of the calculations take the remaining energy at the end of the barrel into consideration. That is to say, when the projectile leaves the rails we assume a fraction f of the current peak is leftover to “discharge” or dissipate in some form or another. To clarify and illustrate this difference, we begin with Figure 2.G.3.

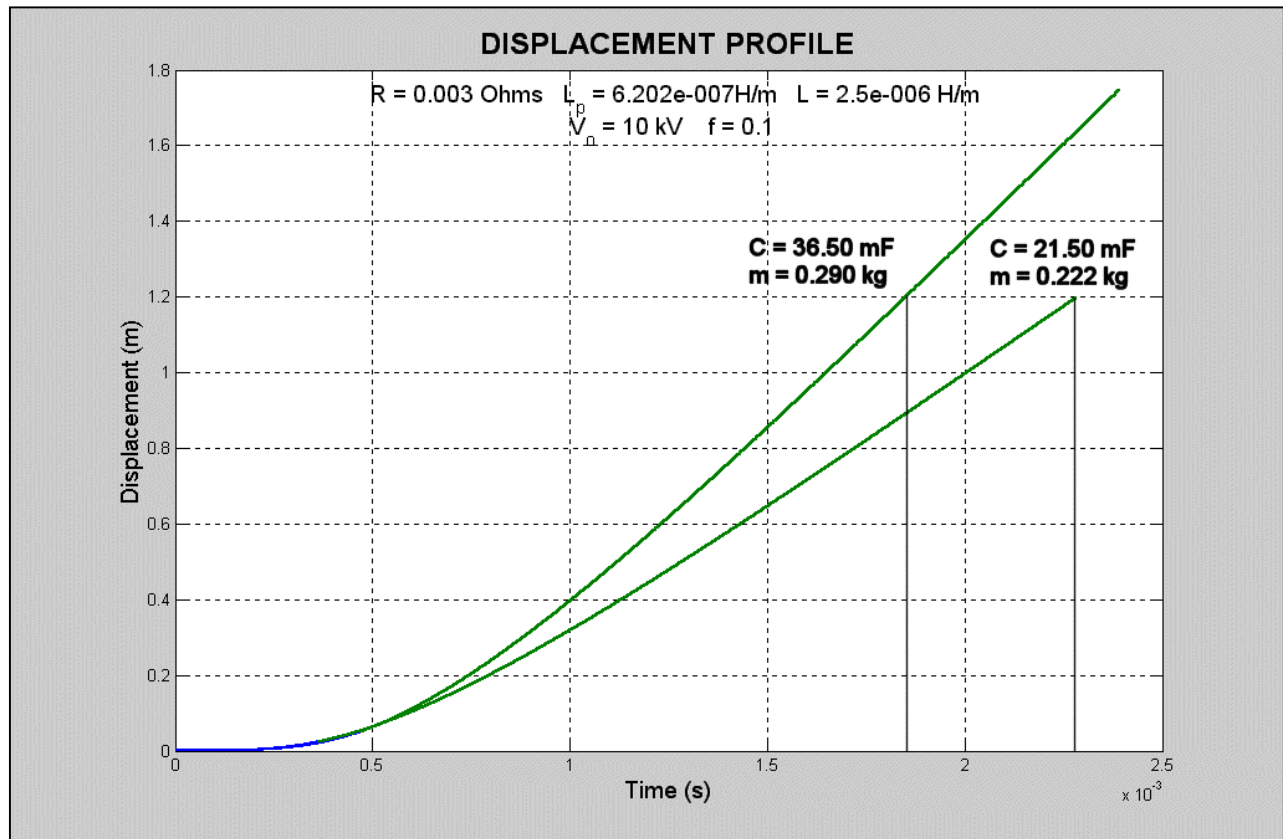


Figure 2.G.3 A comparison of projectile displacement with increased capacitance. With 21.5 mF, $f = 0.1$ at $t = 2.3 \text{ ms}$, and with 36.5 mF, $f = 0.1$ at $t = 2.4 \text{ ms}$.

We can see that our original capacitance of 21.5 mF achieves a 1.2-m projectile displacement in approximately 2.3 ms. However, as we revisit the “interdependency” issues, since there is an increased capacitance, there is an increased projectile acceleration. The projectile will displace more quickly with the increased acceleration. We see from Figure 2.G.3 that an increased capacitance of 36.5 mF displaces the projectile 1.2-m in only 1.7 ms. Now, although there is only a half a millisecond

difference in the displacement, this small difference will correspond to a more significant difference in energy. By using the same time frame, we can now refer to the current pulse profile as shown in Figure 2.G.4.

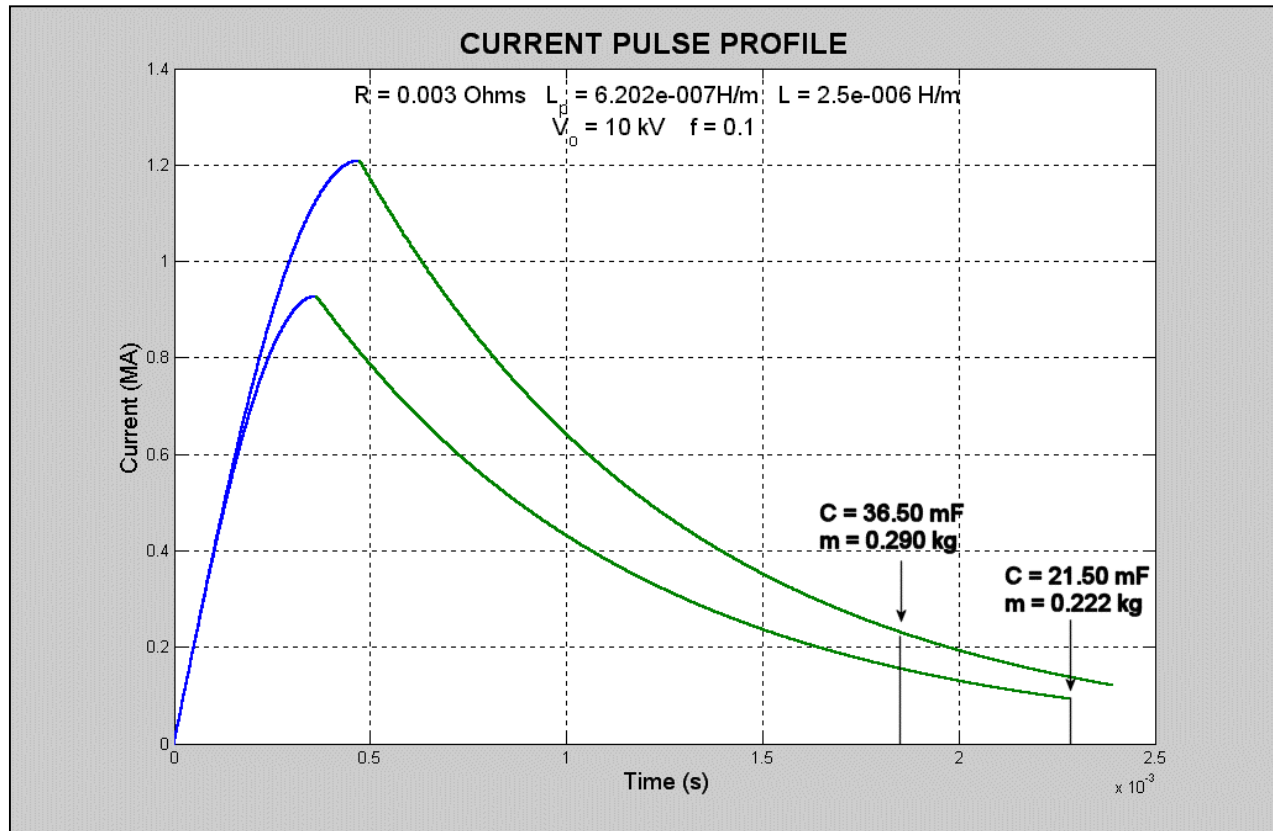


Figure 2.G.4 A comparison of the current pulse with increased capacitance. With 21.5 mF, $f = 0.1$ at $t = 2.3 \text{ ms}$, and 36.5 mF, $f = 0.1$ at $t = 2.4 \text{ ms}$.

From Figure 2.G.4, we can immediately see a difference in remaining current at 1.7 ms and 2.2 ms. At 1.7 ms we would have a residual current of approximately 0.22 MA, while at 2.2 ms we are left with 0.10 MA. Consequently, the 0.12 MA-difference in residual current translates into an energy difference of approximately 36 kJ. Therefore, if a larger capacitance is used to increase the muzzle velocity, 36 more kilo-Joules must either be dissipated by heat, suppressed by a muzzle shunt, or else arc across the rails at the instant the projectile exits the rails. Thus, a change in capacitance may not be the prudent choice to increase the muzzle velocity for a given rail length.

2. Viable Change in Mass

Referring back to Figure 2.G.1, we can achieve a higher muzzle velocity by using a smaller effective mass for a fixed amount of capacitance. For example, if we chose to use 21.5 mF on 0.222 kg projectile we would achieve a muzzle velocity of about 0.735 km/s in a 1.2-m barrel. If we use the same 21.5 mF and a 0.158-kg projectile, we can now achieve a muzzle velocity of about 1.02 kilometer/second. Although we can easily change the mass of the projectile, this still may not be a practical method for increasing the muzzle velocity. To show its practicality, we need look at Figure 2.G.5.

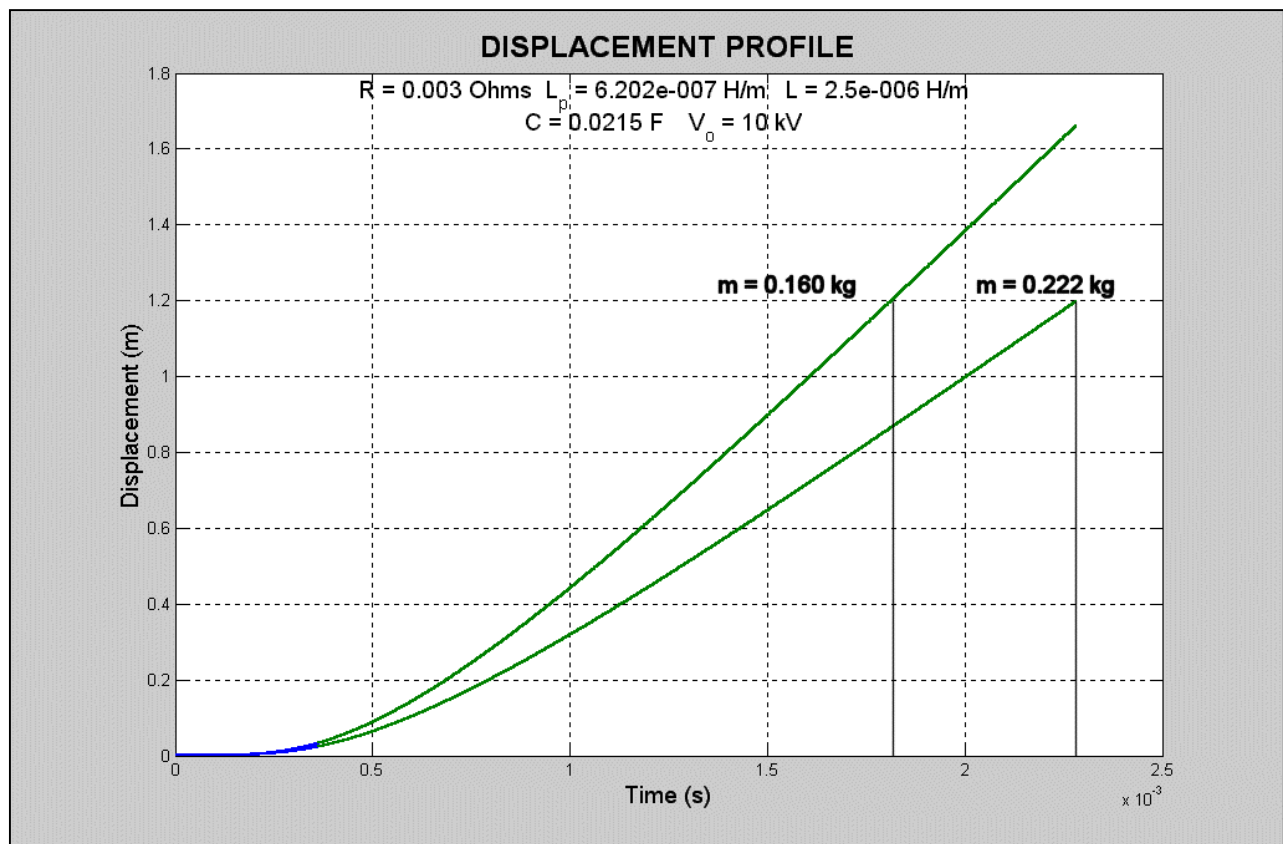


Figure 2.G.5 A comparison of projectile displacement with decreased mass

Again, Figure 2.G.5 shows a difference in time for which the projectile will travel the length of the barrel. A projectile of smaller mass would travel the length of the barrel more quickly than a more massive projectile. Consequently, we would once again expect

to see a difference in the residual current at the end of the rails. Therefore, we now refer Figure 2.G.6.

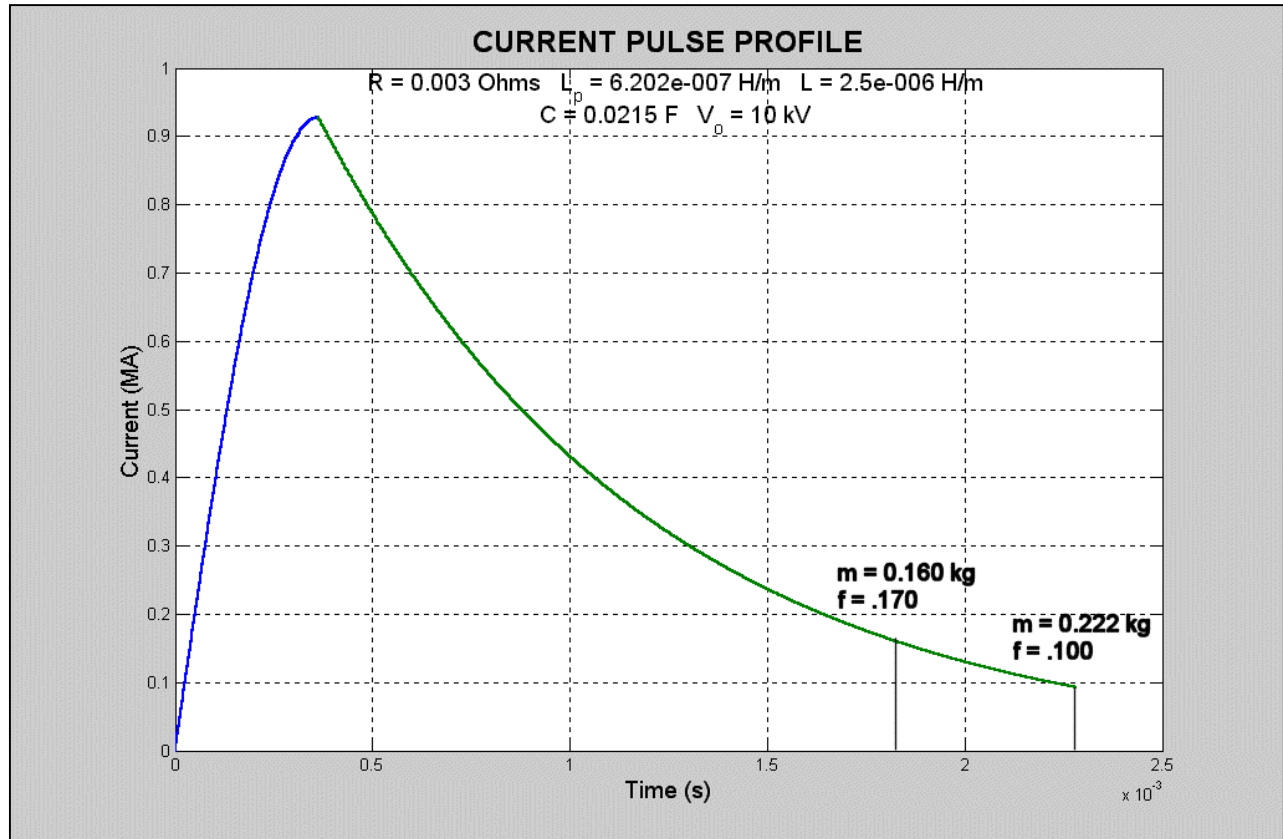


Figure 2.G.6 A comparison of the current pulse with decreased mass

Here, although the mass of the projectile has changed, Figure 2.G.6 shows only one current pulse. This is because unlike the two previous current pulses in Figure 2.G.4, which were dependent on the changing capacitance and subsequent mass, the current pulse as seen Figure 2.G.6 is not dependent on the change in mass. Therefore, it is easier to see the difference in residual current. The 0.222-kg mass exits the barrel with 12.5 kJ of energy ($f = 0.100$) and the 0.160-kg mass exits with 32 kJ ($f = 0.170$). This translates into only 19.5 kJ of extra energy, which is smaller than the 36 kJ in the previous scenario.

H. 1.2-METER RAILGUN SYSTEM

Keep in mind that the purpose of this thesis is to design a power supply for a successful 1.2-m railgun system, where we define “successful” as being able to achieve a muzzle velocity of 1 km/s while effectively using the entire length of barrel. Therefore, if we take into account all of the interdependencies previously discussed, then the system must not only include a sufficient power supply and railgun, but a viable projectile as well. Now, by referring to Figure 2.F.1 we have seen that we could provide a power supply of about 21.5 mF to achieve the following performance profile:

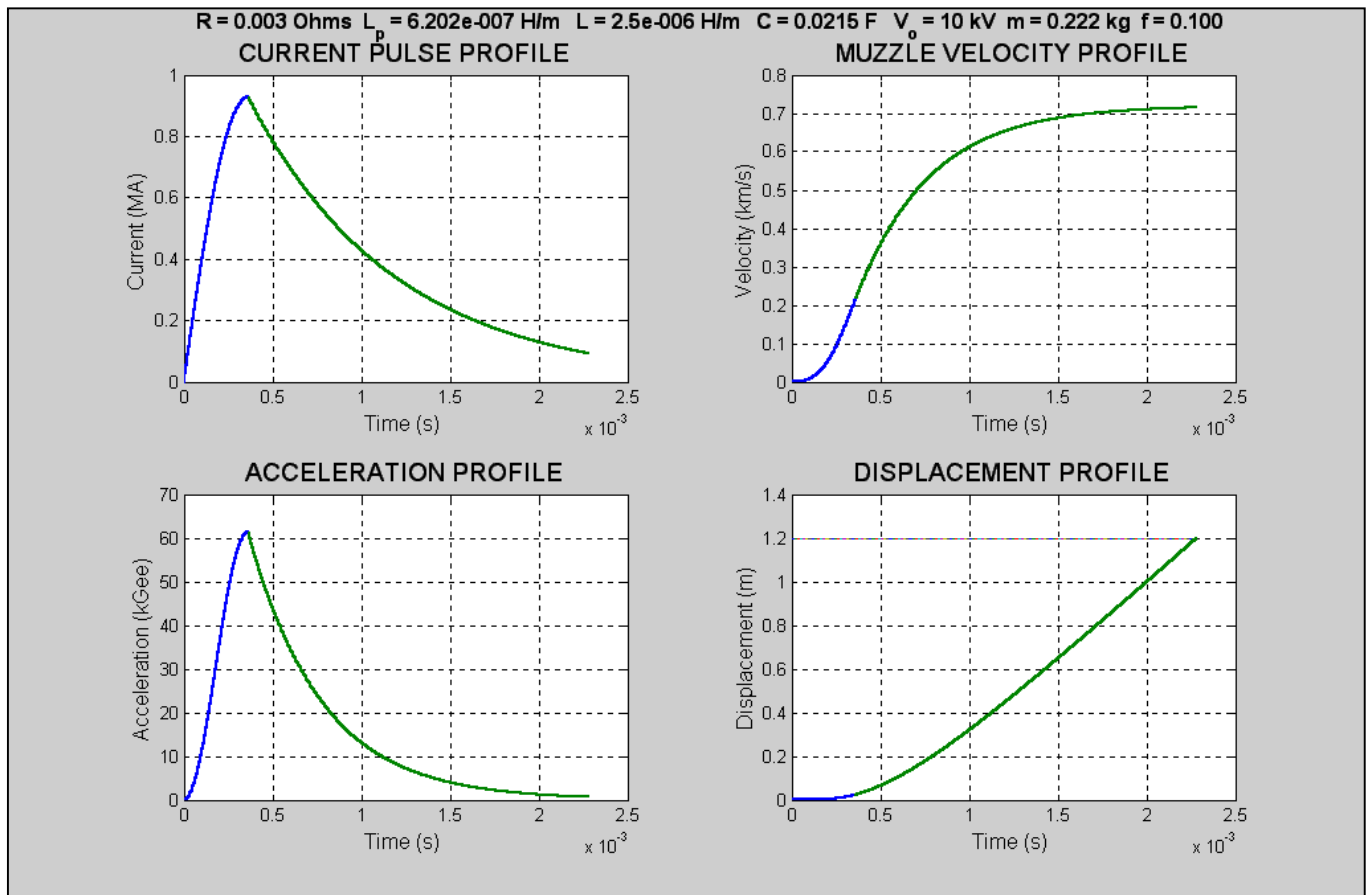


Figure 2.H.1 An optimized railgun system with $m = 0.222 \text{ kg}$ and final $f = 0.100$

Analyzing Figure 2.H.1 shows that if one-tenth of the peak current were left at the end of the rails, the Lorentz force would have effectively displaced the projectile the entire 1.2-m. Unfortunately however, the muzzle velocity would have remained under the desired

goal of 1 km/s. Figure 2.G.1 shows that we can choose a smaller mass for the same amount of capacitance so as to achieve our desired muzzle velocity. Now recall, as previously shown, if we decrease the mass of the projectile we should expect to see a larger fraction f of the peak current left at the end of the rails. So consequently, for a projectile mass of 0.158 kg, we would get the following performance profile

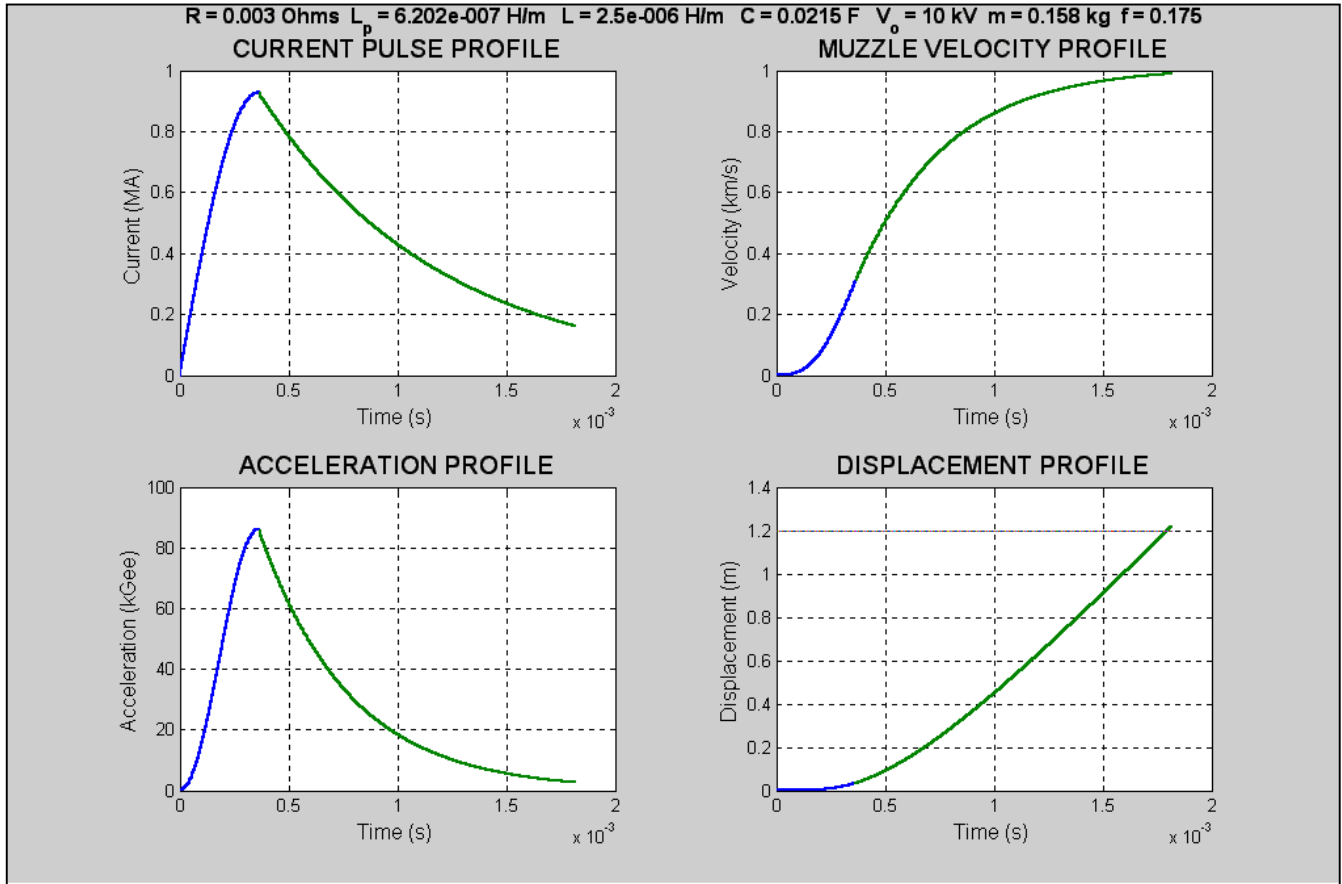


Figure 2.H.2 An optimized railgun system with $m = 0.158 \text{ kg}$ and final $f = 0.175$

As expected, Figure 2.H.2 shows that from the same 21.5 mF power supply, a 0.158-kg projectile would indeed achieve a muzzle velocity of at least 1 km/s. However, as also expected, a larger fraction of the peak current would still be leftover once the projectile exits the barrel. Hence, we must now determine which option is best. In view of that, it may be prudent take a look at the energy trade-offs. By taking the total kinetic energy imparted to the projectile and comparing it to the amount of energy provided to the

breach of the gun, we can calculate an “efficiency” for both options. Equation 2.39 shows the energy relationship in order to calculate the efficiency.

$$\eta = \frac{\frac{1}{2}mv^2}{\frac{1}{2}CV_o^2} \quad (2.39)$$

Where η is the efficiency, m is mass of the projectile, v is final velocity of the projectile, C is the capacitance of the power supply, and V_o is the initial voltage of the power supply.

Figure 2.H.3 below illustrates the results of this premise:

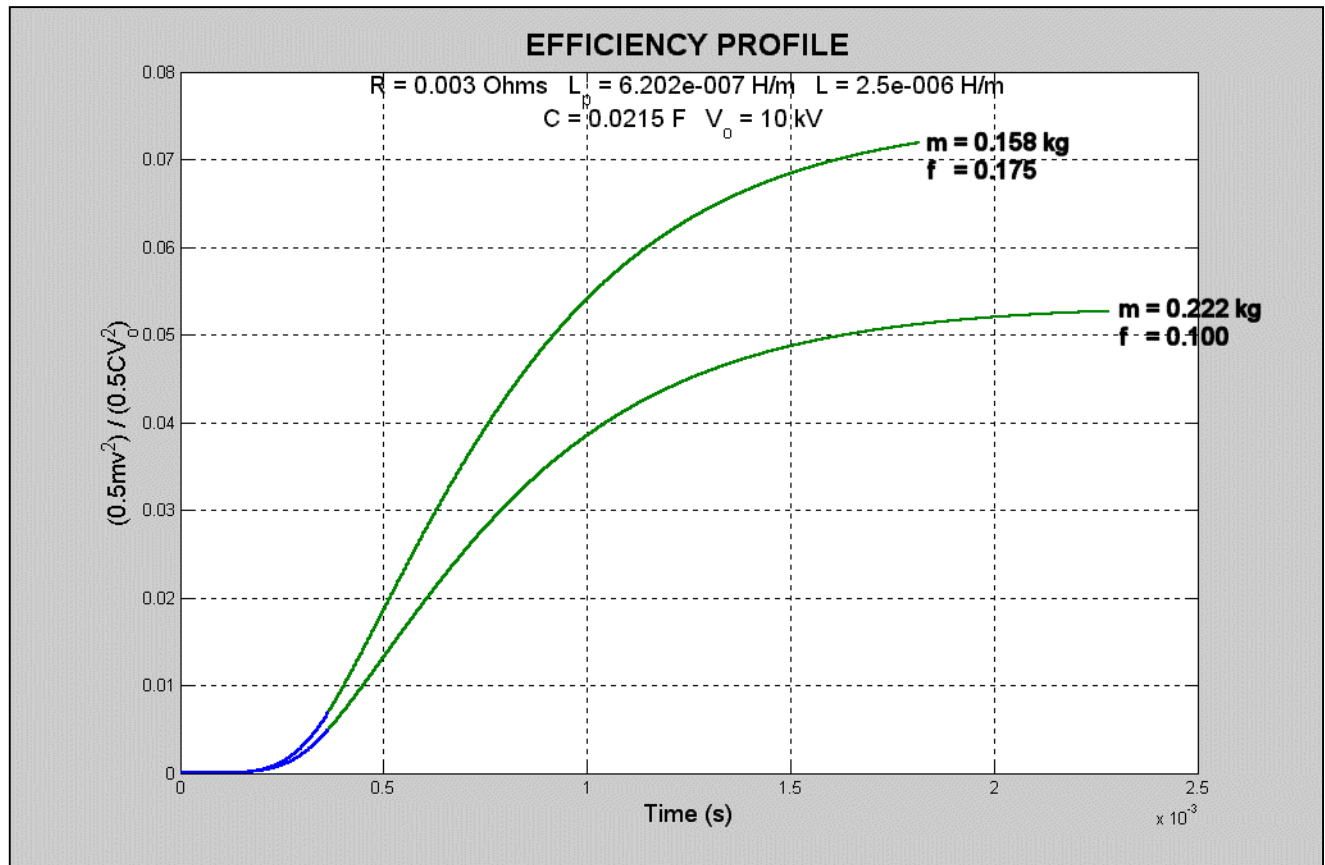


Figure 2.H.3 1.2-meter railgun efficiency versus time

Figure 2.H.3 confirms that choosing a smaller mass gives a larger efficiency over time even though f is larger. This is because although the residual current f left at the end of the rail may differ, Figures 2.H.1 and 2.H.2 show that the current pulse is generally the

same. As a result, the energy provided to the gun breach remains relatively unchanged. Thus, when compared to the kinetic energy out, which is proportional v^2 , the smaller mass would get us “more bang for the buck!” Table 2.H.1 summarizes a successful 1.2-meter railgun system, i.e. $v = 1$ km/s.

RAILGUN	POWER SUPPLY	PROJECTILE
Length: 1.2 m	Capacitance: 7.6 mF	Density: 13.4 g/cm ³ (AgW)
Rail Separation: 0.00625 m	Inductance: 2.50e-6 H/m	Mass: 0.158 kg
L-prime: 0.6202e-6 H/m	Resistance: 3.0 mΩ	
	$f = 0.175$	

Table 2.H.1 Parameters for a 1.2-Meter Naval Postgraduate School Rail Gun System

In summary, Table 2.H.1 shows that by using Lockwood’s 1.2-meter railgun, a minimum 21.5 mF power supply would be required to effectively fire a 0.158-kg Silver-Tungsten projectile at 1 km/s. As a result, about 4% of the initial energy will be dissipated by discharge when the projectile exits or else somehow managed at the end of the rails. Note however, that $\eta \leq 0.07$ and approximately 89% of the initial energy is dissipated as heat prior to the projectile’ exit.

III. A HYPOTHETICAL NAVAL EM RAILGUN

A. 10-METER RAILGUN

In November 2001, the Institute of Advance Technology at Austin, Texas held a workshop to look at the prospect of a naval electromagnetic railgun. As part of a simple scenario, a 10-meter railgun with a characteristic $L' = 0.52 \mu\text{H/m}$ was investigated. The specifics of the study are not yet published. However, we can use our current model to estimate the required capacitance of a power supply, as that shown in Figure 2.A.3, for this hypothetical naval rail gun. But they want a 20 kg projectile and $V_0 = 12 \text{ kV}$.

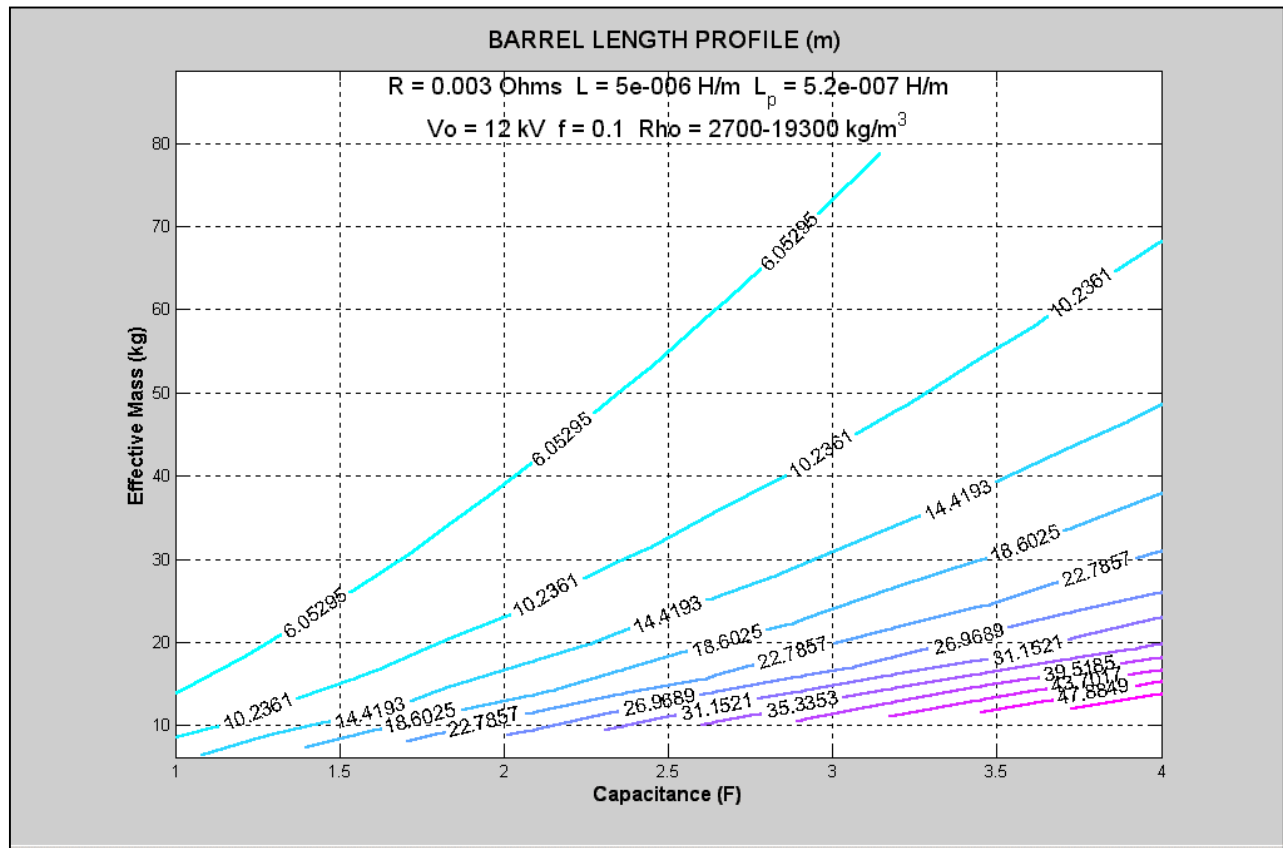


Figure 3.A.1 Naval railgun lengths as a function of mass and capacitance

Again, assuming a 10-meter barrel and by referring to Table 1.B.1, Figure 3.A.1 shows that a 60-kg projectile would require a power supply consisting of approximately 3.7 F of capacitance. This capacitance would take up considerable volume on a ship.

B. MUZZLE VELOCITY OF A 10-METER RAILGUN

The muzzle velocity profile over the same range of mass and capacitance is as follows:

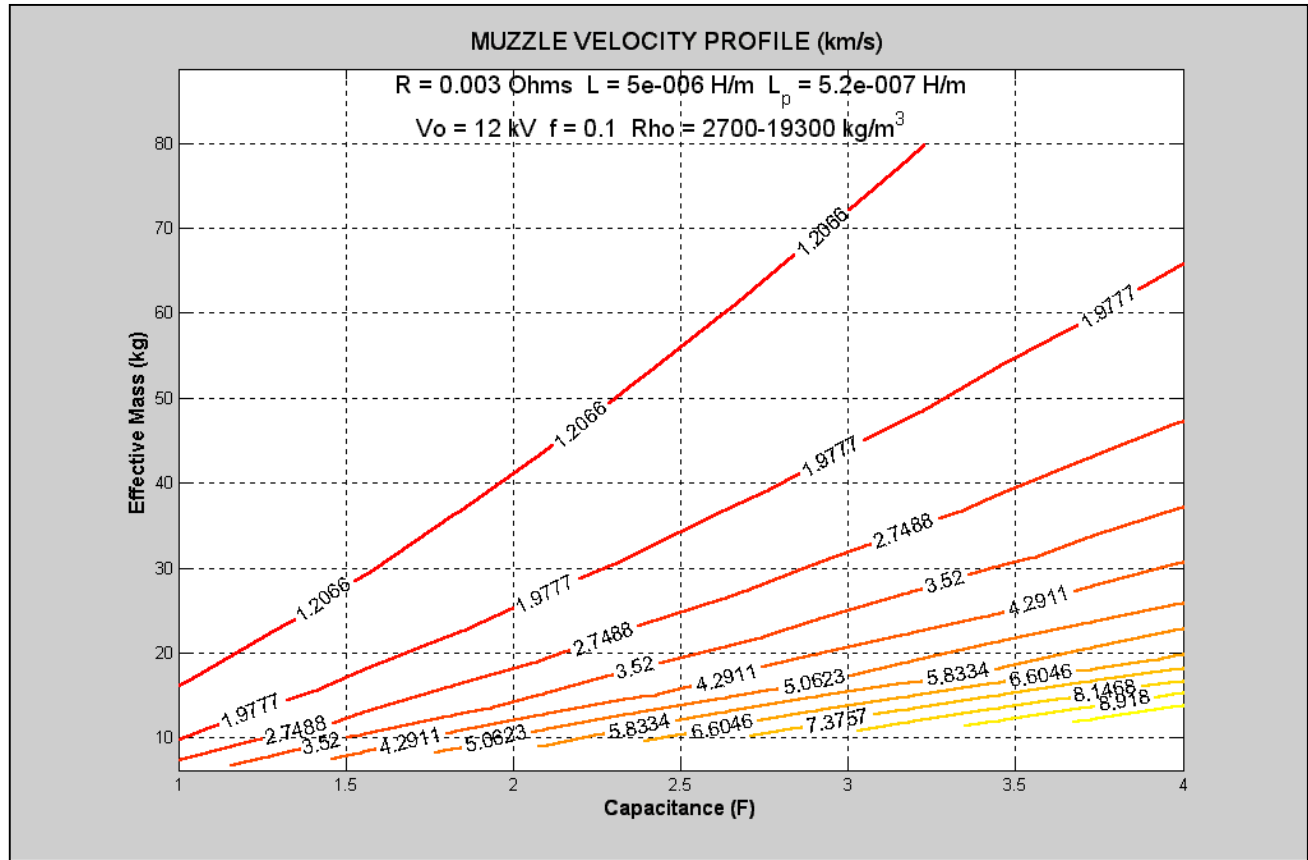


Figure 3.B.1 Naval railgun muzzle velocities as a function of mass and capacitance

Figure 3.B.1 shows that a 3.7-F power supply would only accelerate a 60-kg projectile to approximately 1.98 km/s. However, as it was shown previously in section 2.G.2, we can achieve a higher muzzle velocity by reducing the mass, but at the cost of a higher residual current. Therefore, consider a 44-kg projectile instead of a 60-kg projectile. Figure 3.B.1 shows that the same 3.7 F power supply would accelerate the 44-kg projectile to about 2.5 km/s. To take a closer look, we refer to the curves in Figure 3.B.2.

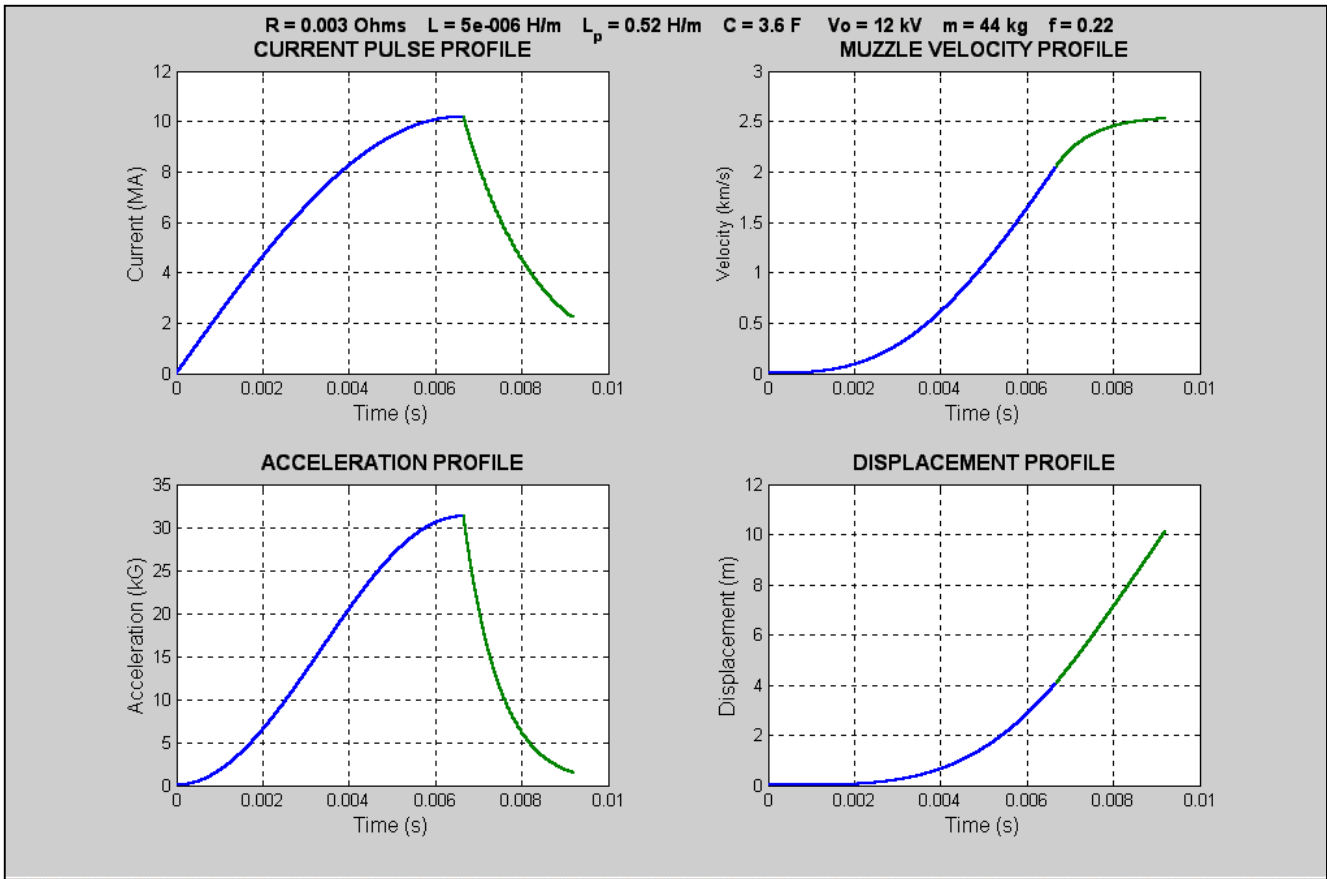


Figure 3.B.2 10-meter Naval Railgun System Profile

Figure 3.B.2 shows that the performance for a 44-kg projectile, in the same 10-m barrel and 3.7 F scenario, would indeed achieve a muzzle velocity of 2.5 km/s. Furthermore, the 10-m railgun would suffer approximately 4.5% of the initial capacitive energy at the end of the rails. Even so, at the cost of a little mass, we were able to achieve a higher muzzle velocity and perhaps a still lethal projectile.

C. LETHAL KINETIC ENERGY

As mentioned earlier in the introduction to this thesis, one advantage of a naval railgun would be its lethality. Projectiles traveling at hyper-velocities on the order of a kilometer per second would produce an enormous amount of damage to a target. To get a feel for the magnitude of kinetic energy that a projectile would have, we refer to Figure 3.C.1.

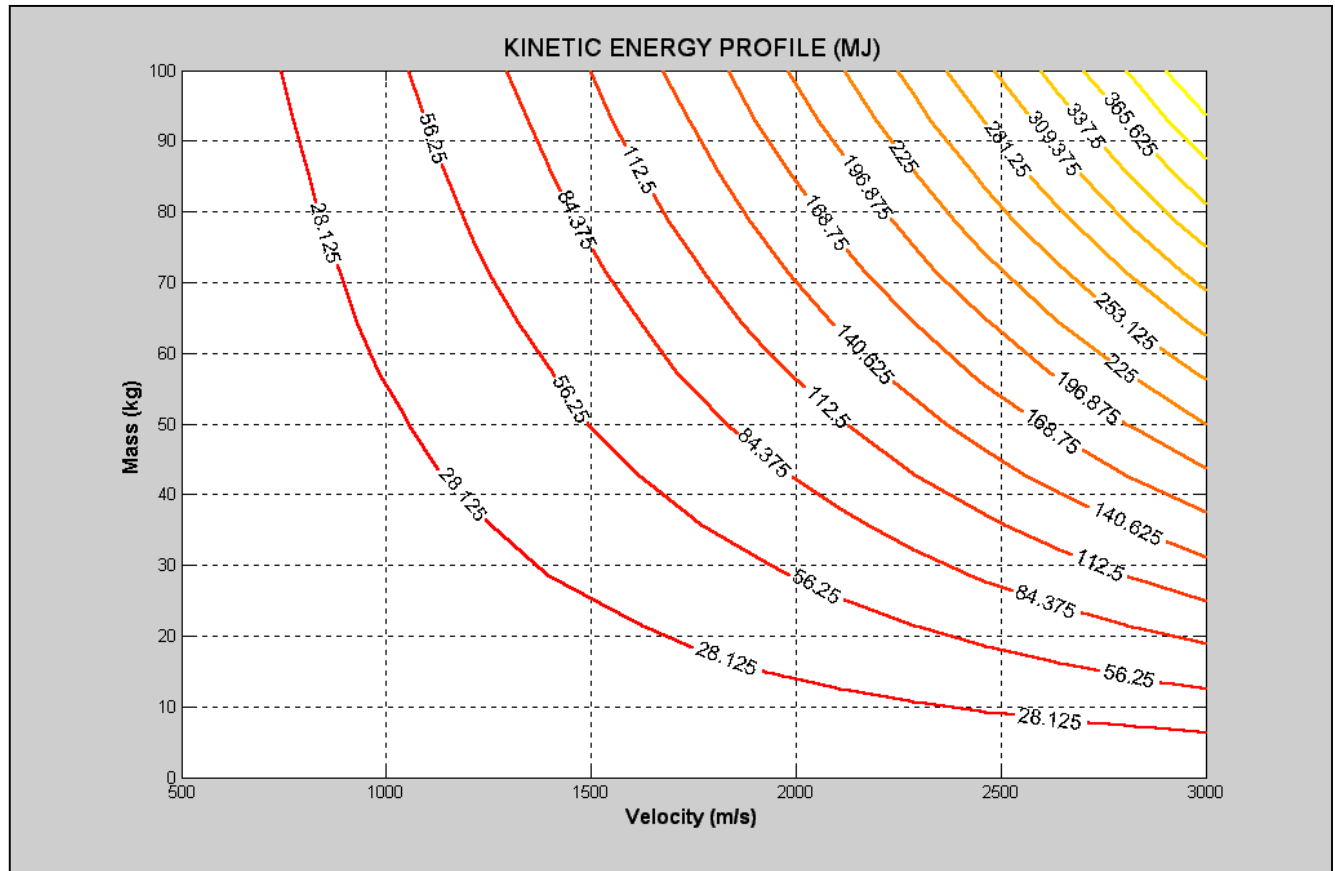


Figure 3.C.1 Kinetic Energy Profile for a Naval Railgun Projectile

Referring to the previous section and Figure 3.C.1, a 60-kilogram projectile traveling at 1.98 km/s would have a kinetic energy of about 118 MJ. This energy is comparable to approximately 28 kg of TNT. But a 44-kg projectile traveling at 2.5 km/s would have a kinetic energy about 138 MJ. This energy is comparable to approximately 33 kg of TNT. Therefore, the mass reduction in the previous section not only achieved a higher muzzle velocity, but a more lethal projectile as well.

In summary, we now have a reasonable prediction for a full-scale naval electromagnetic railgun. We can specify an expected barrel length and determine the amount capacitance required to effectively use that barrel. We can also determine the muzzle velocity as a function of capacitance and projectile mass. The only task that remains is to build one.

IV. CONCLUSION

The objective of this thesis was to propose a viable power supply for the existing 1.2-meter railgun at the Naval Postgraduate School. By referencing Equation 1.9, a MATLAB model was created to show the direct relationship between the current pulse and the Lorentz force exerted on a projectile. As a result, the performance of an ideal railgun could reasonably be predicted. From fixed initial parameters, (maximum current density, circuit inductance, and barrel length) the effects of railgun geometry, capacitance, residual current, contact surface, projectile mass and projectile mass density on muzzle velocity, acceleration and kinetic energy were examined. Figures 2.F.1 and 2.G.1 showed that tradeoffs, such as using a smaller mass for a greater muzzle velocity, can be made to achieve desired performances. However, because of the interdependencies between several factors, other consequences may result, such as a larger residual current. A recommended power supply having 21.5 mF capacitance appeared to be suitable for the 1.2-meter railgun. Hence, although the power supply could have been independently constructed from the railgun and projectile, our model has shown that an effective railgun system must be considered as a whole. For the 1.2-m railgun, 21.5 mF should fire a 158-g projectile at about 1 km/s.

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APPENDIX (A) MATLAB CURRENT MODEL AND PREDICTED RAILGUN PERFORMANCE

```

% ALLAN FELICIANO
% RAILGUN THESIS
% September 01, 2001
% CURRENT versus TIME and
% FORCE DISPLACEMENT versus TIME and
% PROJECTILE VELOCITY versus TIME and
% PROJECTILE ACCELERATION versus TIME

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% vo          Initial velocity of projectile (m/s)          %
% xo          Initial displacement of the projectile        %
% L_prime     Permeabilty constant for augmented railgun (H/m) %
% Voltage     Initial Voltage (V)                          %
% R           Characteristic resistance of railgun circuit (Ohms) %
% L           Characteristic inductance of the railgun circuit (H/m) %
% C           Capacitance (F)                              %
% Jmax        Maximum current density allowed              %
% height      Separation between the rails                 %
% f           Fraction of the Peak Current Io. (A)         %
% Io          Maximum/Peak Current (A)                    %
% A           Contact Surface Area of projectile (m2)      %
% I_cap       Current due to capacitor discharge (A)       %
% I_induc     Current due to fall off (A)                  %
% vf         Muzzle velocity                               %
% xf         Final dispalcement of force (i.e.)length of rails (m) %
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear

xo = 0;          % Meters
vo = 0;          % Meters/Second
L_prime = 6.202E-7; % Henries/meter
Voltage = 10000; % Volts
R = 0.0030;      % Ohms
L = 2.5E-6;      % Henries
C = 21.50E-3;    % Farads
Jmax = 350E6;    % Amps/meter^2
height = 0.00625; % Meter
f = 0.175;       % Fraction of Peak Current
rho = 13400;     % Kilograms/meter^3 (Silver-tungsten)
pi = 3.141592654;

str1 = num2str(R);
str2 = num2str(L);
str3 = num2str(C,8);
str4 = num2str(Voltage);
str5 = num2str(f);
str7 = num2str(L_prime);

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                               TIME SEGMENTS                               %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

t_prime = pi*sqrt(L*C)/2          % Time for capacitor discharge
tf       = -(L/R)*log(f)          % Time for inductor discharge

t_cap = 0:0.000001:t_prime;      % 0 to t-prime
t_induc = t_prime:0.000001:(t_prime+tf); % t-prime to t-final

time = t_prime + tf;              % Total time

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                               CURRENT PEAK                               %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

Io = Voltage * sqrt( C/L );       % Peak Current in Amps

A = Io/Jmax;                      % Surface Area (Meter^2)

m = rho*A*height;                % Projectile Mass (kilograms)
str6 = num2str(m);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                               CURRENT PULSE                               %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

w = 1 / sqrt(L*C);               % Frequency (Hertz)

I_cap = Io*sin(w*t_cap);          % Current due to Capacitor Discharge

I_induc = Io*exp( -(R/L)*(t_induc - t_prime) ); % Current due to Fall Off

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                               PROJECTILE VELOCITY (Within the Rails)      %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

accel = L_prime*(Io^2)/(4*m);     % Acceleration Factor

part1 = sin(2*w*t_cap);
part2 = 2*w;
vf1 = accel*(t_cap - (part1/part2)); % Velocity Due to Capacitor Discharge

part3 = accel*(t_prime);
part4 = exp( -2*R*(t_induc - t_prime)/L );
part5 = accel*(L/R)*(1 - part4);
vf2 = part3 + part5;              % Velocity Due to Fall Off

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                               FORCE DISPLACEMENT (Within the Rails)        %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

element1 = cos(2*w*t_cap)/(4*(w^2));
element2 = 1 / (4*(w^2));
element3 = (t_cap.^2)/2;
xf1 = accel*(element3 - (element2 - element1)); % Displacement Due to Capacitor Discharge

```

```

elem1 = cos(2*w*t_prime)/(4*(w^2));
elem2 = 1 / (4*(w^2));
elem3 = (t_prime^2)/2;
x_prime = accel*( elem3 - (elem2 - elem1) );

time2 = t_induc - t_prime;
element4 = t_prime*time2;
element5 = (L/R)*time2;
element6 = 0.5*((L/R)^2)*(part4 - 1);
xf2 = x_prime + accel*(element4 + element5 + element6); % Displacement Due to Fall Off

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                               PROJECTILE ACCELERATION (Within the Rails)                               %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

acceleration1 = accel*((sin(w*t_cap)).^2); % Acceleration Due to Capacitor Discharge

acceleration2 = accel*(exp(-2*R*(t_induc - t_prime)/L )); % Acceleration Due to Fall Off

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                               PLOTS                               %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Current
figure(1)
gcf = plot(t_cap,(I_cap/1E6),t_induc,(I_induc/1E6)),grid
title(['CURRENT PULSE PROFILE', ' R = ',str1,' Ohms'...
' L_prime = ',str7,' H/m', ' L = ',str2,' H/m', ' C = ',str3,' F'...
' Vo = ',str4,' V', ' m = ',str6,' kg', ' f = ',str5])
xlabel('Time (s)'), ylabel('Current (MA)')
set(gcf,'LineWidth',1.5)
hold on

%Muzzle Velocity
figure(2)
gcf = plot(t_cap,(vf1/1000),t_induc,(vf2/1000)), grid
title(['MUZZLE VELOCITY PROFILE', ' R = ',str1,' Ohms'...
' L_prime = ',str7,' H/m', ' L = ',str2,' H/m', ' C = ',str3,' F'...
' Vo = ',str4,' V', ' m = ',str6,' kg', ' f = ',str5])
xlabel('Time (s)'), ylabel('Velocity (km/s)')
set(gcf,'LineWidth',1.5)
hold on

%Barrel Length
figure(3)
gcf = plot(t_cap,xf1,t_induc,xf2), grid
title(['FORCE DISPLACEMENT PROFILE', ' R = ',str1,' Ohms'...
' L_prime = ',str7,' H/m', ' L = ',str2,' H/m', ' C = ',str3,' F'...
' Vo = ',str4,' V', ' m = ',str6,' kg', ' f = ',str5])
xlabel('Time (s)'), ylabel('Displacement (m)')
set(gcf,'LineWidth',1.5)
hold on

%Acceleration
figure(4)

```

```

gcf = plot(t_cap,(acceleration1/9800),t_induc,(acceleration2/9800)),grid
title(['ACCELERATION PROFILE', ' R = ',str1,' Ohms'...
' L_prime = ',str7, 'H/m', ' L = ',str2, 'H/m', ' C = ',str3, ' F'...
' Vo = ',str4, ' V', ' m = ',str6, ' kg', ' f = ',str5])
xlabel('Time (s)'), ylabel('Acceleration (kGee)')
set(gcf,'LineWidth',1.5)
hold on

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                               COMPARISON PLOTS                               %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

figure(5)

%Current
subplot(2,2,1)
gcf = plot(t_cap,(I_cap/1E6),t_induc,(I_induc/1E6)),grid
title(['CURRENT PULSE PROFILE', ' R = ',str1,' Ohms'...
' L_prime = ',str7, 'H/m', ' L = ',str2, 'H/m', ' C = ',str3, ' F'...
' Vo = ',str4, ' V', ' m = ',str6, ' kg', ' f = ',str5])
xlabel('Time (s)'), ylabel('Current (MA)')
set(gcf,'LineWidth',1.5)
hold on

%Muzzle Velocity
subplot(2,2,2)
gcf = plot(t_cap,(vf1/1000),t_induc,(vf2/1000)), grid
title(['MUZZLE VELOCITY PROFILE', ' R = ',str1,' Ohms'...
' L_prime = ',str7, 'H/m', ' L = ',str2, 'H/m', ' C = ',str3, ' F'...
' Vo = ',str4, ' V', ' m = ',str6, ' kg', ' f = ',str5])
xlabel('Time (s)'), ylabel('Velocity (km/s)')
set(gcf,'LineWidth',1.5)
hold on

%Barrel Length
subplot(2,2,4)
gcf = plot(t_cap,xf1,t_induc,xf2), grid
title(['FORCE DISPLACEMENT PROFILE', ' R = ',str1,' Ohms'...
' L_prime = ',str7, 'H/m', ' L = ',str2, 'H/m', ' C = ',str3, ' F'...
' Vo = ',str4, ' V', ' m = ',str6, ' kg', ' f = ',str5])
xlabel('Time (s)'), ylabel('Displacement (m)')
set(gcf,'LineWidth',1.5)
hold on

%Acceleration
subplot(2,2,3)
gcf = plot(t_cap,(acceleration1/9800),t_induc,(acceleration2/9800)),grid
title(['ACCELERATION PROFILE', ' R = ',str1,' Ohms'...
' L_prime = ',str7, 'H/m', ' L = ',str2, 'H/m', ' C = ',str3, ' F'...
' Vo = ',str4, ' V', ' m = ',str6, ' kg', ' f = ',str5])
xlabel('Time (s)'), ylabel('Acceleration (kGee)')
set(gcf,'LineWidth',1.5)
hold on

```

APPENDIX (B) MATLAB MODEL FOR BARREL LENGTH AND MUZZLE VELOCITY

```

% BARREL LENGTH versus CAPACITANCE and
% MUZZLE VELOCITY versus CAPACITANCE

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% vo          Initial velocity of projectile (m/s)          %
% xo          Initial displacement of the projectile        %
% L_prime     Permeability constant for augmented railgun (H/m) %
% Voltage     Initial Voltage (V)                          %
% R           Characteristic resistance of railgun circuit (Ohms) %
% L           Characteristic inductance of the railgun circuit (H/m) %
% C           Capacitance (F)                              %
% Jmax        Maximum current density allowed              %
% height      Separation between the rails                 %
% f           Fraction of the Peak Current (Io). (A)       %
% Io          Maximum/Peak Current (A)                     %
% A           Contact Surface Area of projectile (m2)      %
% vf          Muzzle velocity                              %
% xf          Final displacement of force (i.e.)length of rails (m) %
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear

xo      = 0;          % Meters
vo      = 0;          % Meters/Second
L_prime = 6.202E-7;   % Henries/meter
Voltage = 10000;      % Volts
R       = 0.0030;     % Ohms
L       = 2.5E-6;     % Henries
Jmax    = 350E6;      % Amps/meter^2
height  = 0.00625;    % Meter
f       = 0.10;       % Fraction of Peak Current
pi      = 3.141592654;

points = 20;

C = linspace(1.660E-3, 8.000E-3, points); % Farads
rho = linspace(9000, 13400, points);      % Kilograms

% Create a grid
% Assigns values of Capacitance as X and Mass as Y
[X,Y] = meshgrid(C,rho);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                               CURRENT PEAK                               %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

Io = Voltage * sqrt( X/L );

%Minimum Surface Area required for each C and maximum current density Jmax
A = ( Io/Jmax );

```

```

%Projectile volume from each A and fixed height
volume = A*height;

%Projectile's effective mass
m = Y.*A*height;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                               TIME SEGMENTS                               %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Capacitor discharge time
t_prime = (pi/2) * sqrt( L*X );

%Inductive Energy xfer time
tf = -(L/R) * log(f);

%Total time
time = t_prime + tf;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                               ACCELERATION FACTOR and Parts                               %
%                               (Not to be confused with actual projectile acceleration)                               %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

accel_1 = (L_prime*(Jmax^2))*(A.^2);

accel_2 = 4*m;

accel = accel_1./accel_2;

Part1 = 0.7335 * ( L*X );

Part2 = ( (L/R)^2 ) * ( -log(f) - ((1-(f^2))/2) );

Part3 = (L/R) * (1-(f^2));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                               FINAL FORCE DISPLACEMENT                               %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

xf = vo*(t_prime + tf) + accel.*( Part1 + (t_prime*tf) ) + Part2 );

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                               PROJECTILE MUZZLE VELOCITY                               %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

vf = vo + accel.*(t_prime + Part3);

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                PLOTS                                %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```

%FORCE DISPLACEMENT (Barrel Length)
figure(1)
[LABEL,h] = contour(X,m,xf,points)
clabel(LABEL,h)
colormap(cool)
j = findobj('Type','patch');
set(j,'LineWidth',1.5)
grid
title(['BARREL LENGTH PROFILE (meters)', ' R = ',str1,' Ohms'...
' L = ',str2,' H/m', ' L_p = ',str3,' H/m'...
' Vo = ',str4,' kV', ' f = ',str5,...
' Rho = ',str7,'-',str8,' kg/m^3'])
xlabel('Capacitance (F)'), ylabel('Effective Mass (kg)')

```

```

%MUZZLE VELOCITY
figure(2)
[LABEL,h] = contour(X,m,(vf/1000),points)
clabel(LABEL,h)
colormap(autumn)
j = findobj('Type','patch');
set(j,'LineWidth',1.5)
grid
title(['MUZZLE VELOCITY PROFILE (km/s)', ' R = ',str1,' Ohms'...
' L = ',str2,' H/m', ' L_p = ',str3,' H/m'...
' Vo = ',str4,' kV', ' f = ',str5,...
' Rho = ',str7,'-',str8,' kg/m^3'])
xlabel('Capacitance (F)'),
ylabel('Effective Mass (kg)')

```

```

%PROJECTILE SURFACE AREA
figure(4)
subplot(1,2,1)
gcf = plot(C,Area),grid
title(['MINIMUM SURFACE AREA (m^2)', ' R = ',str1,' Ohms'...
' L = ',str2,' H/m', ' L_p = ',str3,' H/m'...
' Vo = ',str4,' kV', ' f = ',str5,...
' Rho = ',str7,'-',str8,' kg/m^3'])
xlabel('Capacitance (F)'),
ylabel('Surface Area (m^2)')
set(gcf,'LineWidth',1.5)

```

```

%PROJECTILE MASS
subplot(1,2,2)
gcf = plot(C,mass),grid
title('PROJECTILE MASS (kg)')
xlabel('Capacitance (F)'),
ylabel('Mass (kg)')
set(gcf,'LineWidth',1.5)

```

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